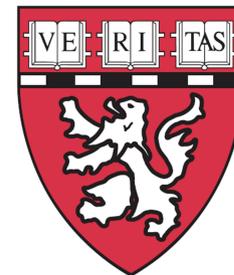


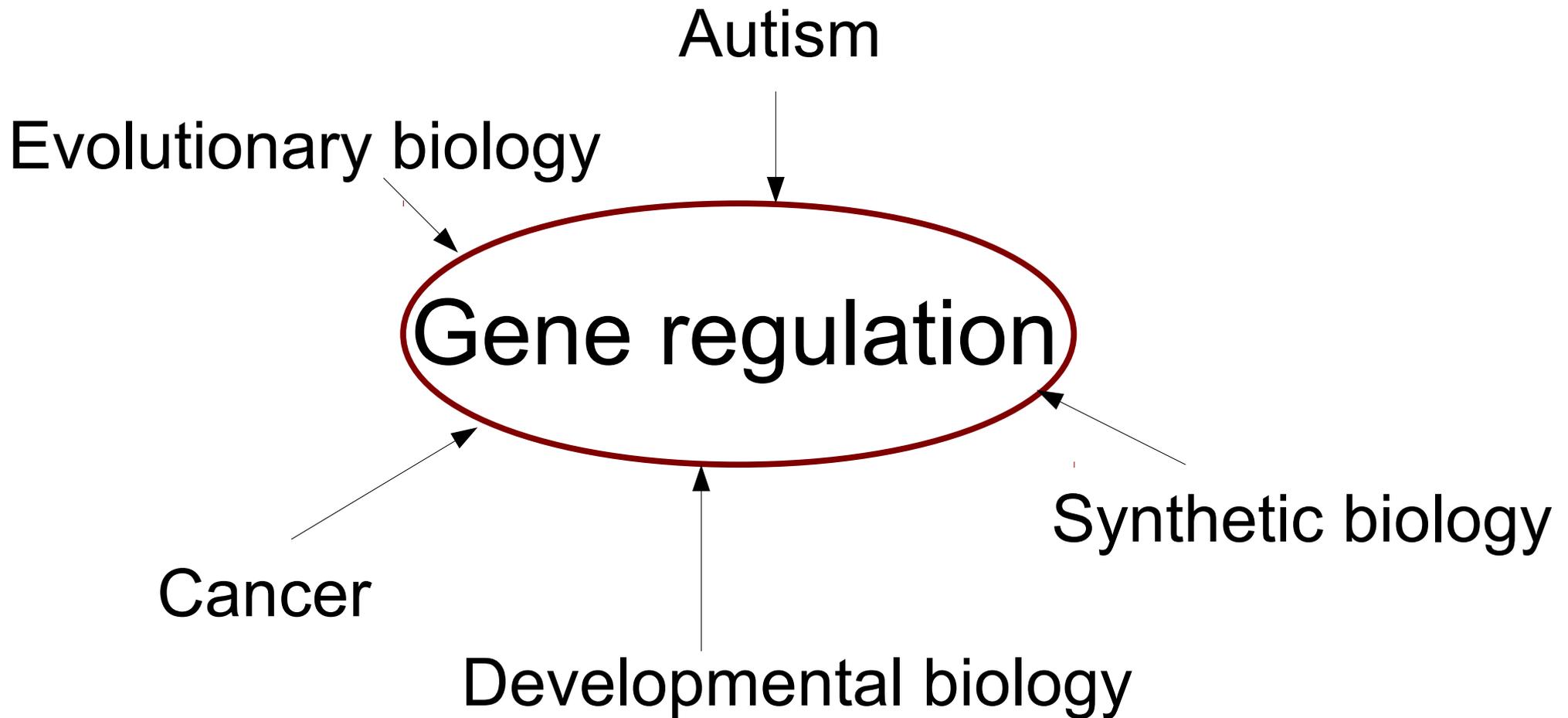
# Models of distal enhancers of inducible gene expression

Martin Hemberg

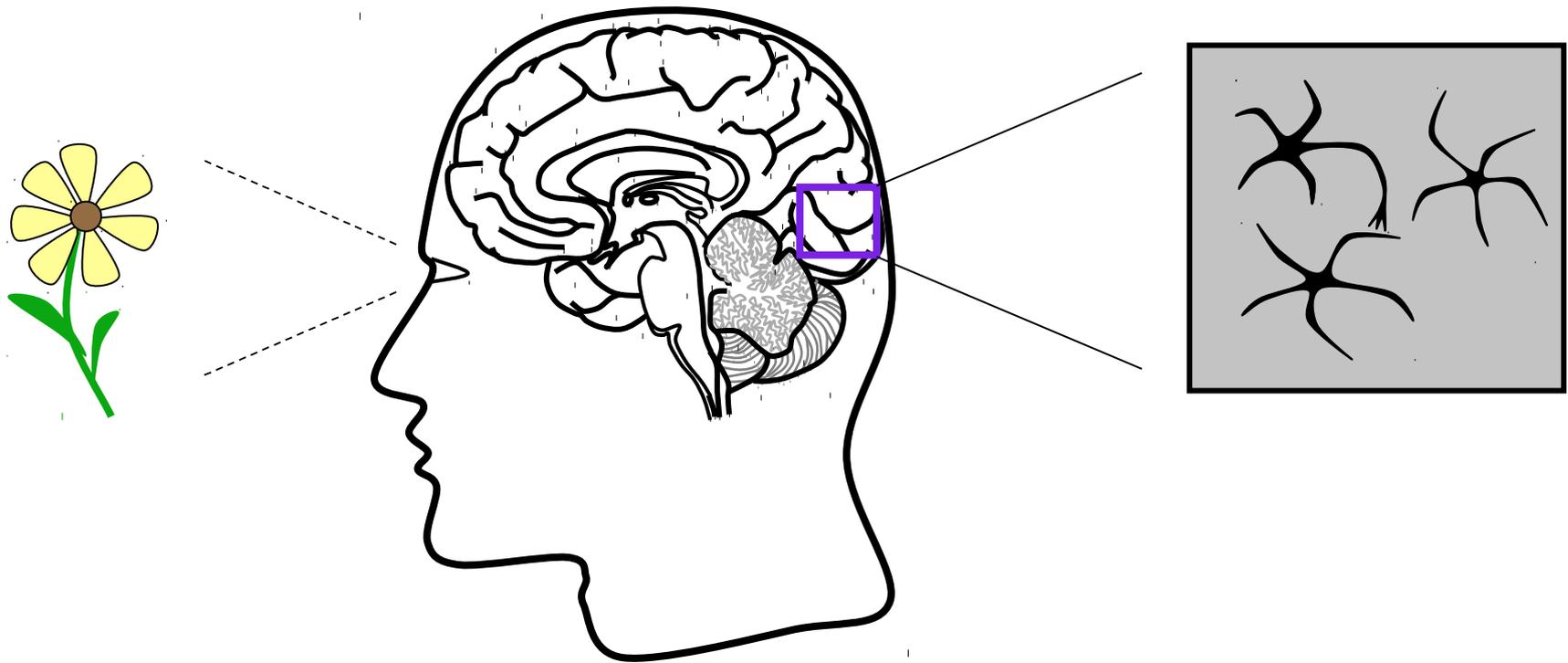
Channing Division of Network Medicine  
November 30, 2012



# Why is gene regulation important?

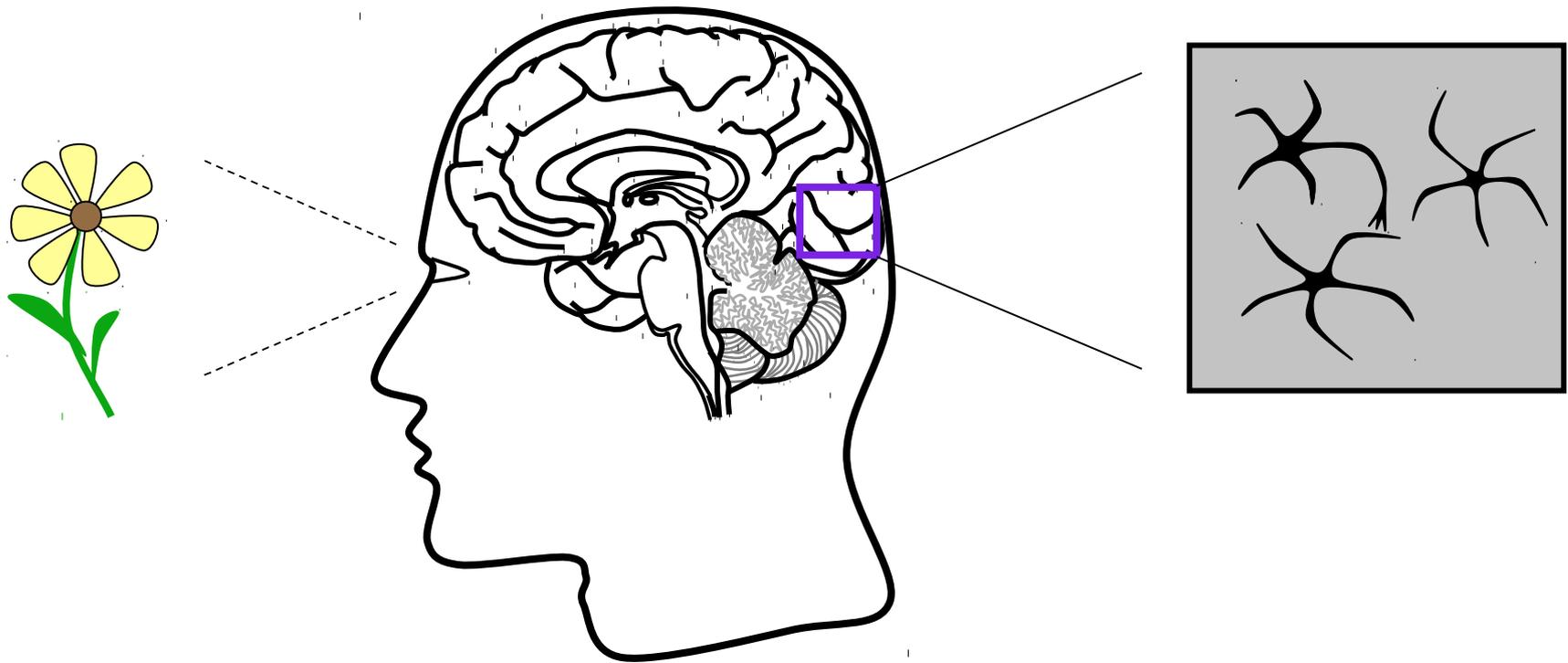


# Synapses change in response to external environmental stimuli



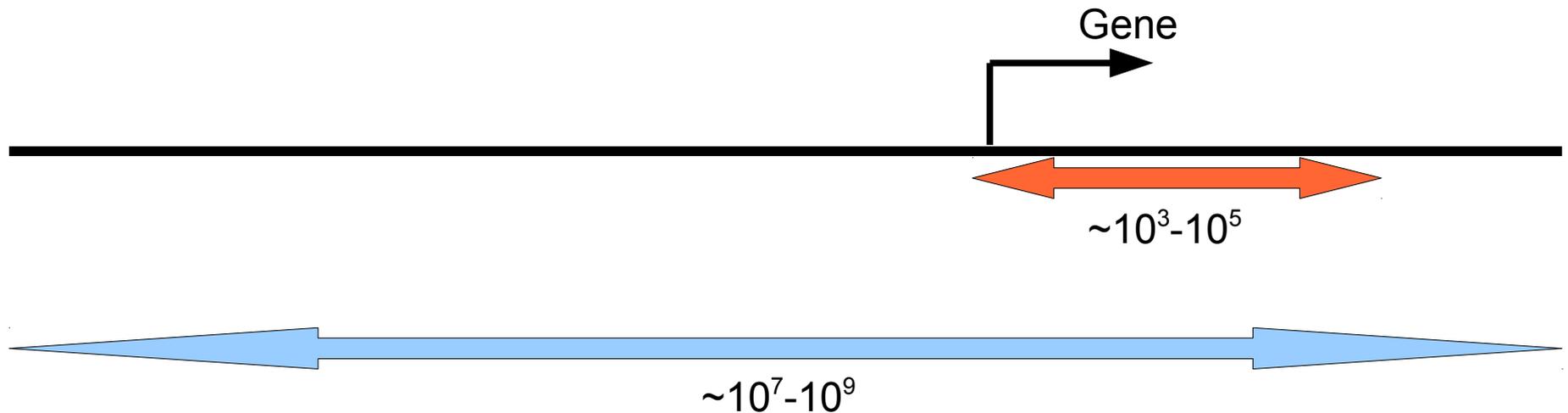
Hubel & Wiesel, 1970's

# Activity dependent gene expression triggered by influx of Calcium ions



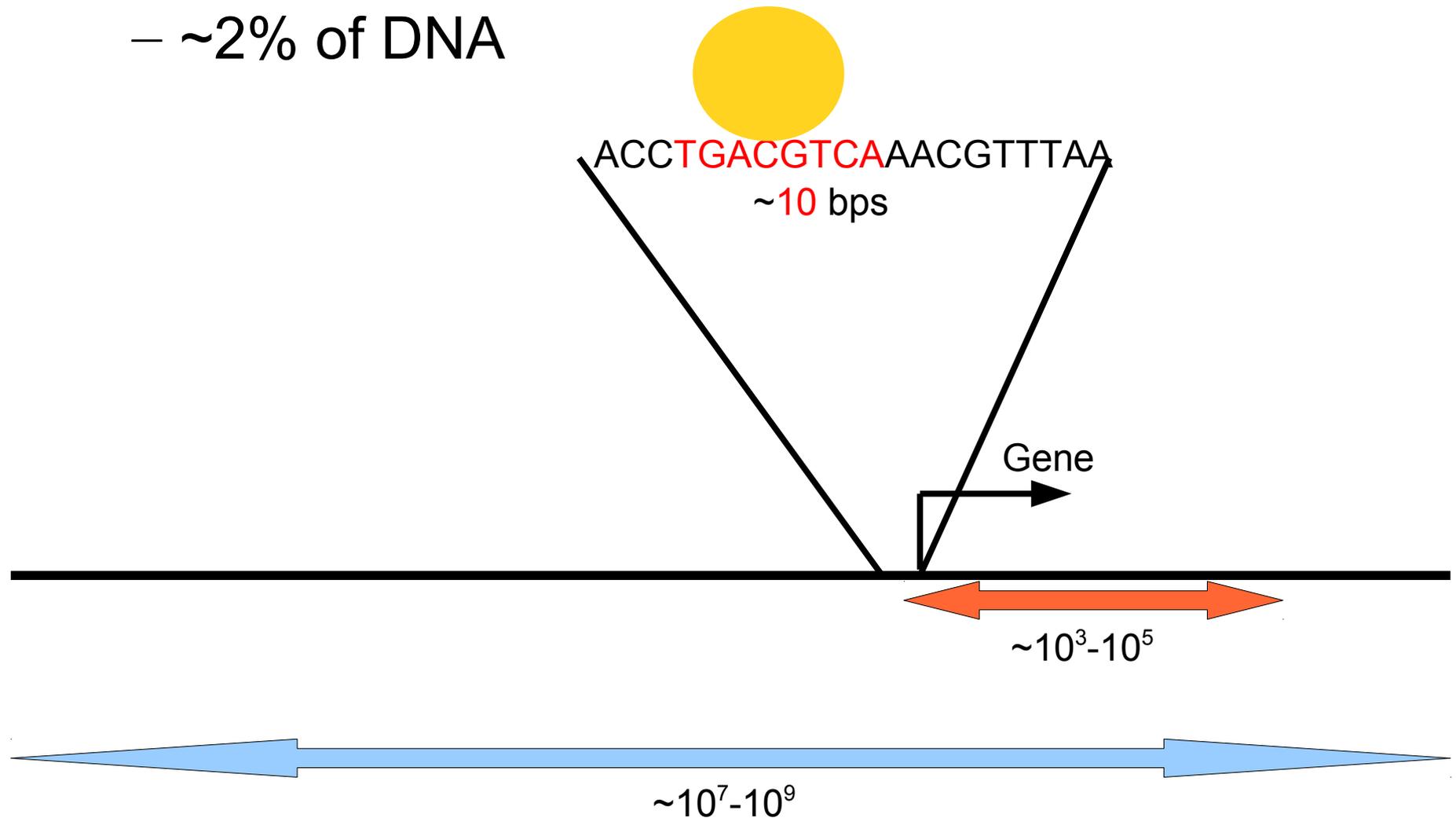
Mouse genome has ~3 billion bps

- ~25,000 genes
  - ~2% of DNA



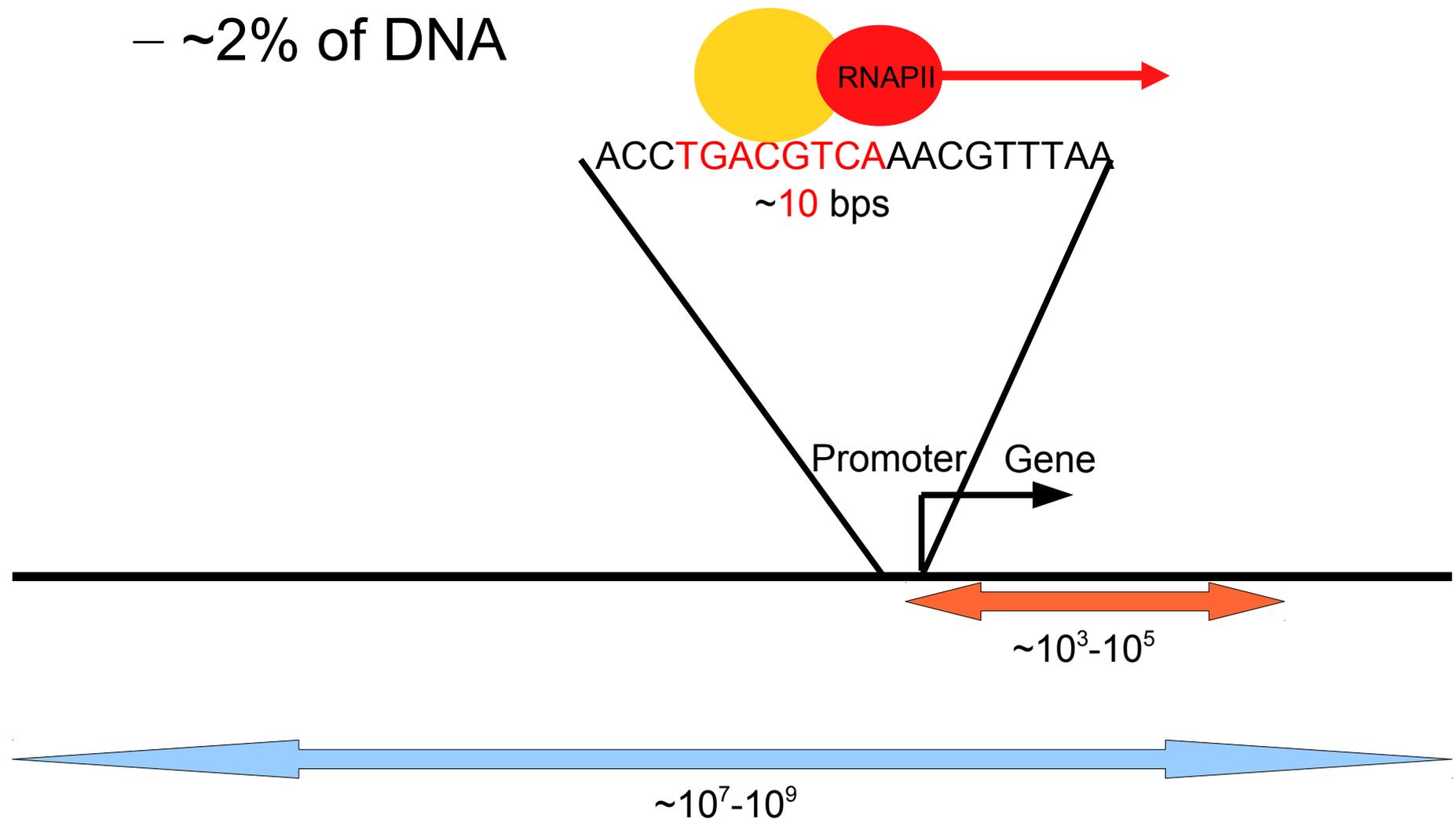
# Transcription Factors (TFs) bind to DNA

- ~25,000 genes
  - ~2% of DNA

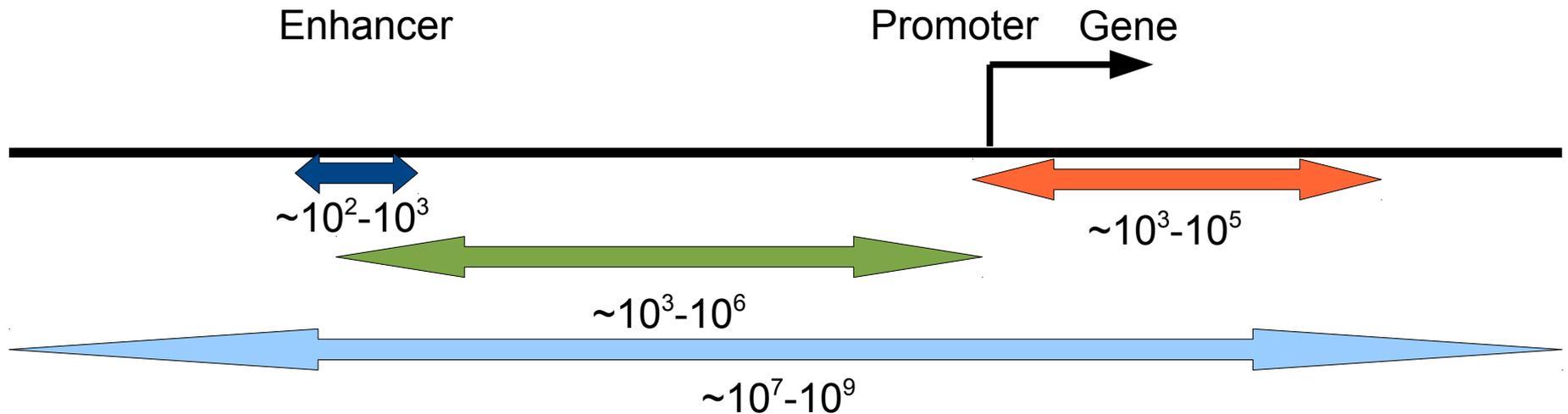


# Transcription factors bind at promoter to recruit RNA Polymerase II (RNAPII)

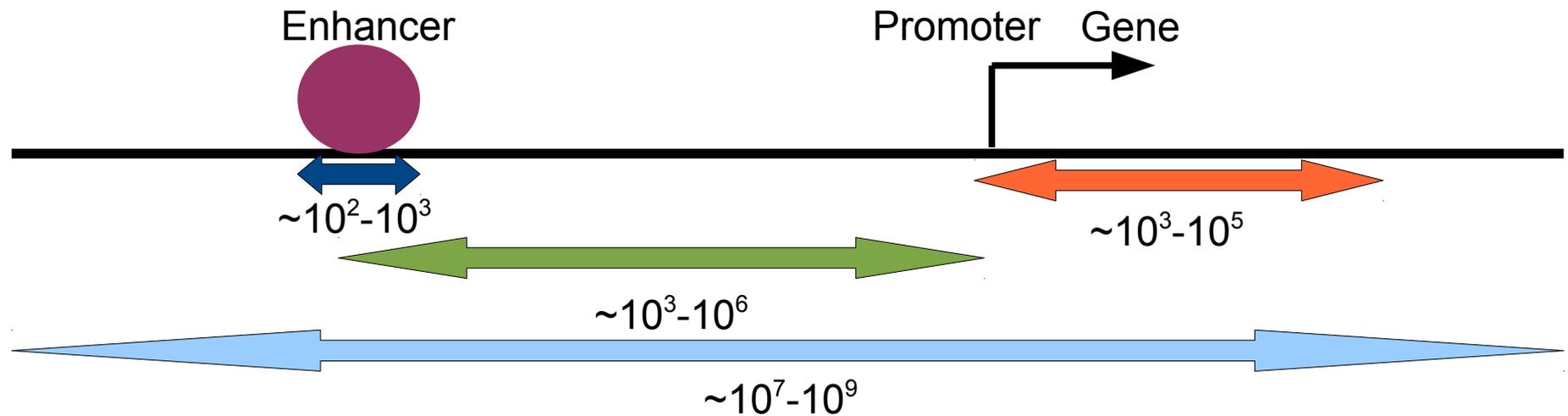
- ~25,000 genes
  - ~2% of DNA



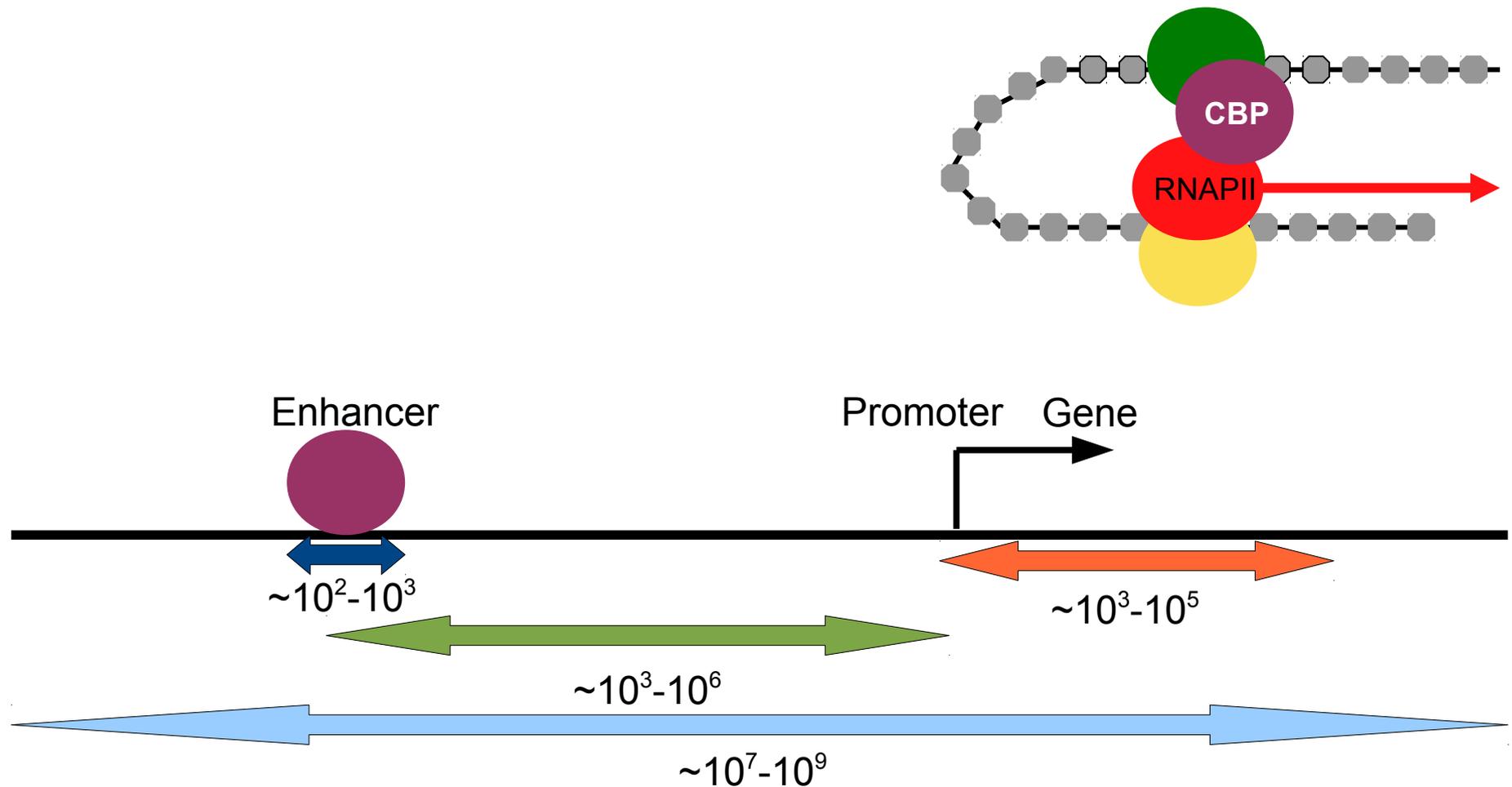
# Enhancers are distal regulatory sequences



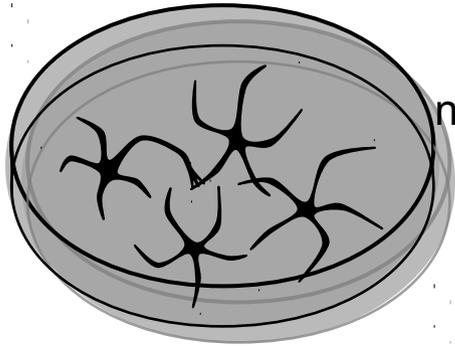
# Enhancers characterized by **CBP** binding



# The mechanism by which enhancers increase expression are poorly understood



# Cultured mouse cortical neurons for genome-wide study of activity dependent gene expression

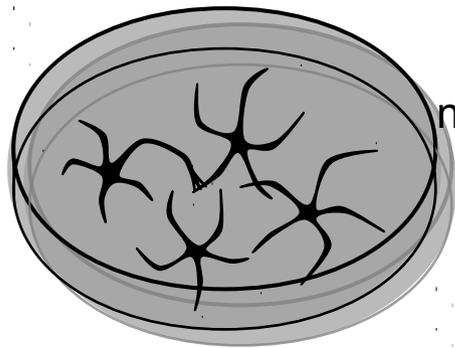


mouse cortical  
neurons

neuronal activation via potassium chloride (**KCl**) depolarization



# Genome-wide data obtained using high-throughput sequencing



mouse cortical neurons

neuronal activation via potassium chloride (**KCl**) depolarization

- KCl

+ KCl

ChIP-Seq  
RNA-Seq

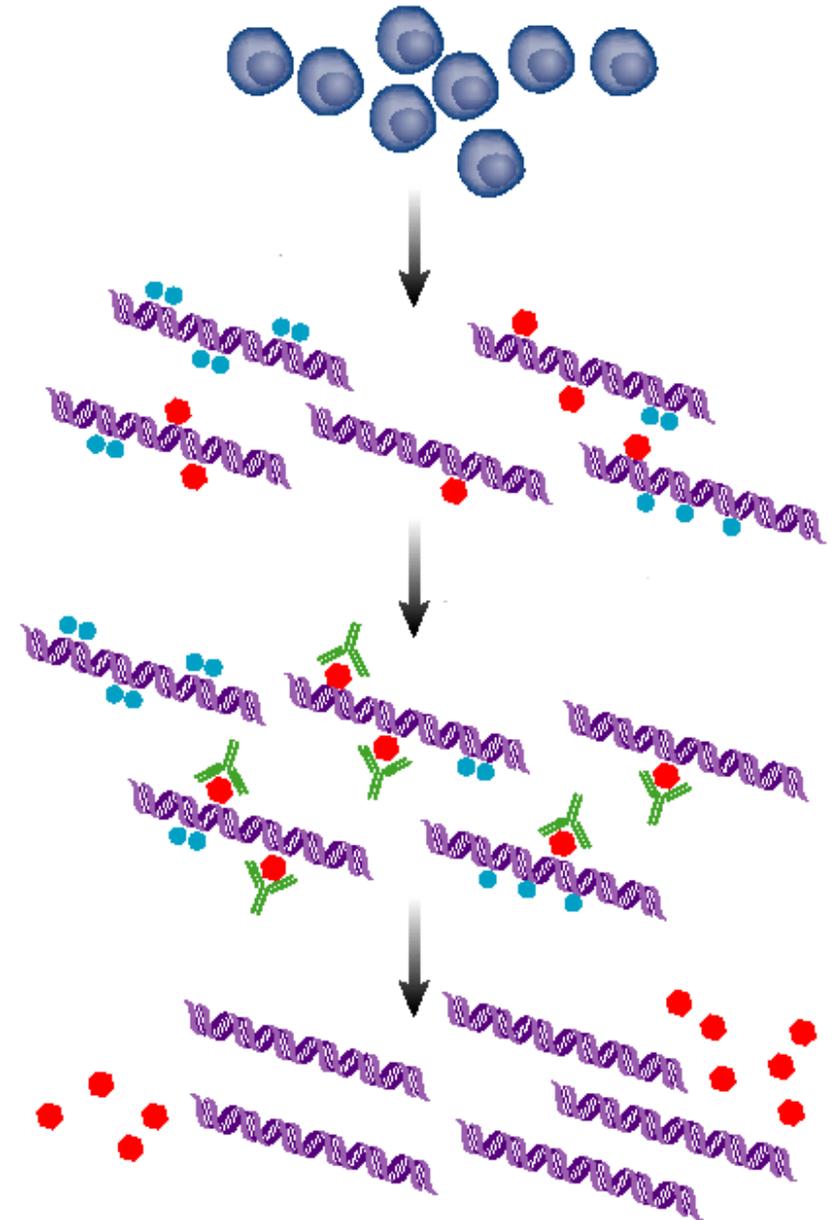
ChIP-Seq  
RNA-Seq



Jesse Gray  
Tae-Kyung Kim  
Greenberg Lab

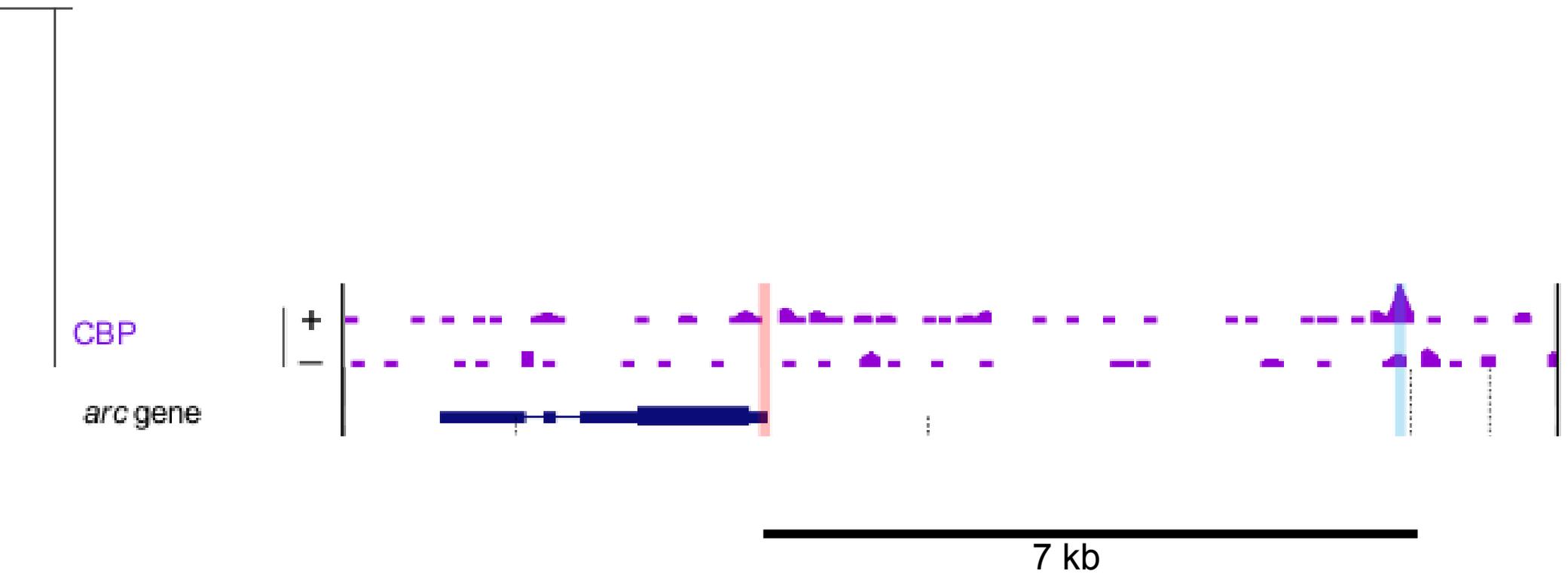
# Chromatin immunoprecipitation and sequencing (**ChIP-Seq**) finds protein binding sites *in vivo*

- Short **reads** mapped to reference genome
- #reads ~ binding
- $\sim 10^6$  reads
- Unbiased

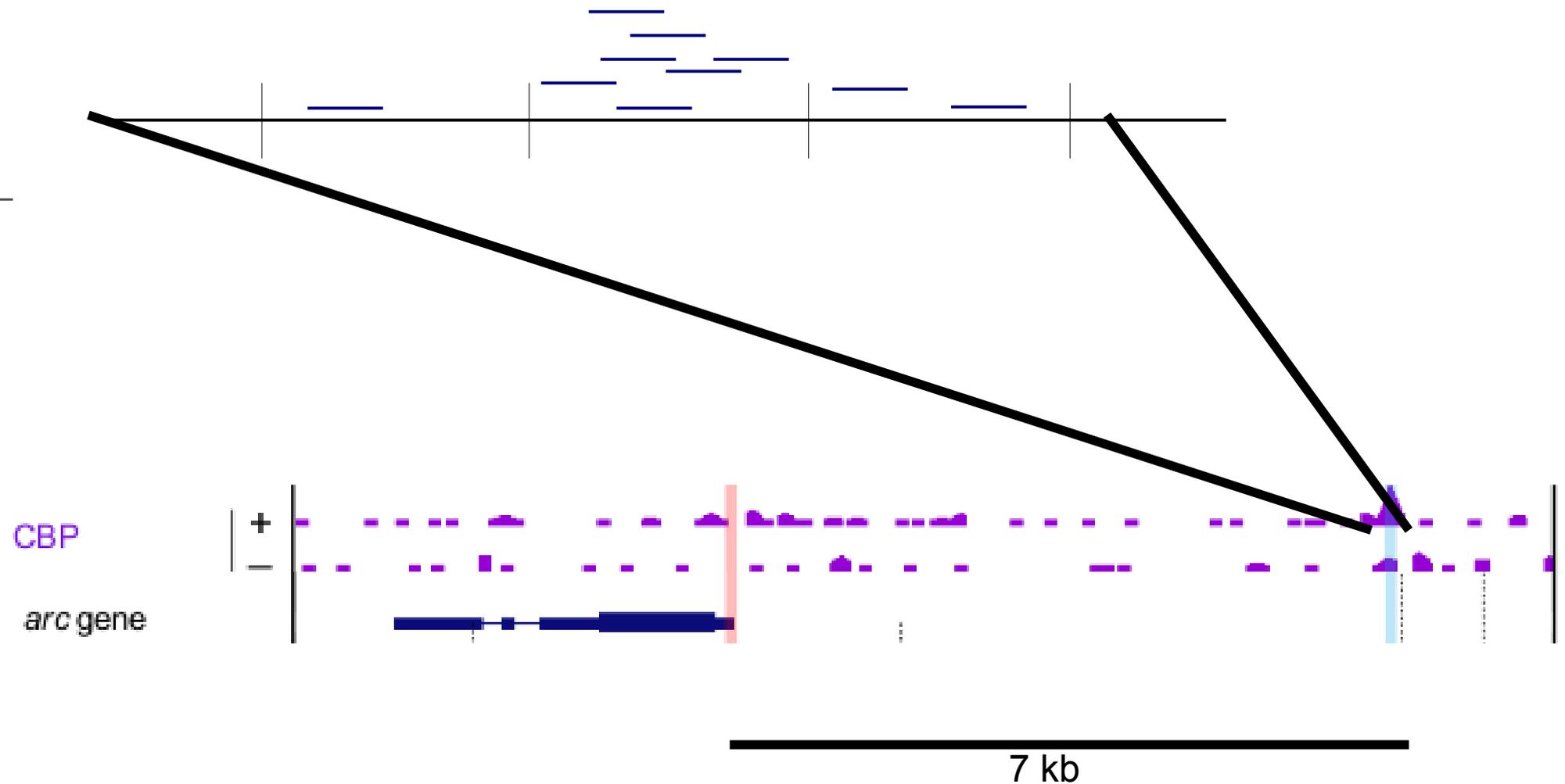


(Mardis, 2007)

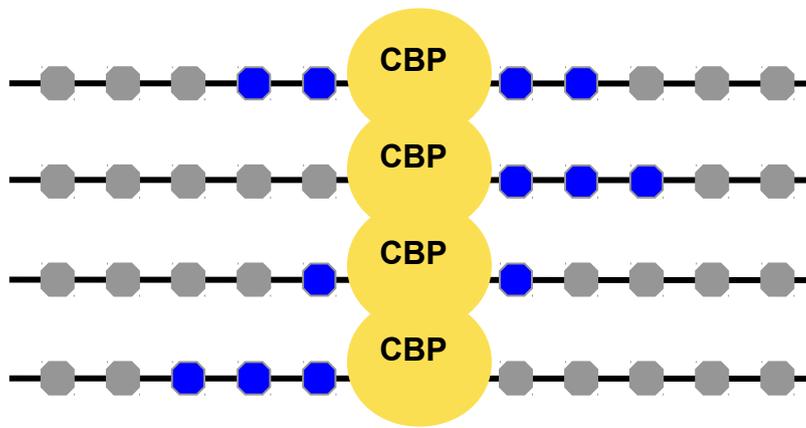
# Inducible CBP binding at enhancers



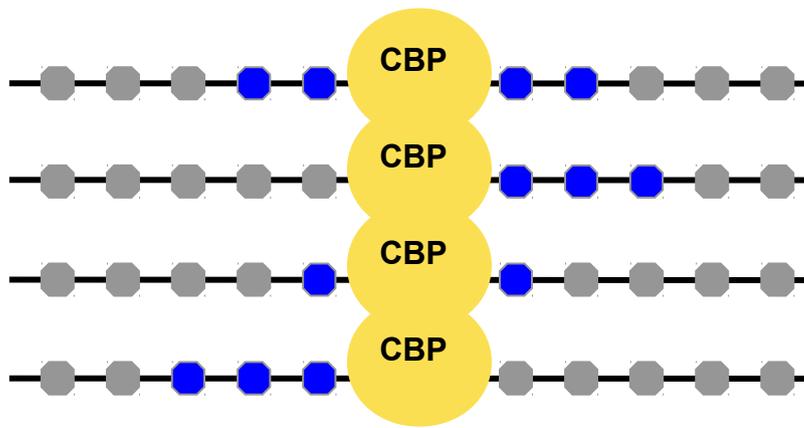
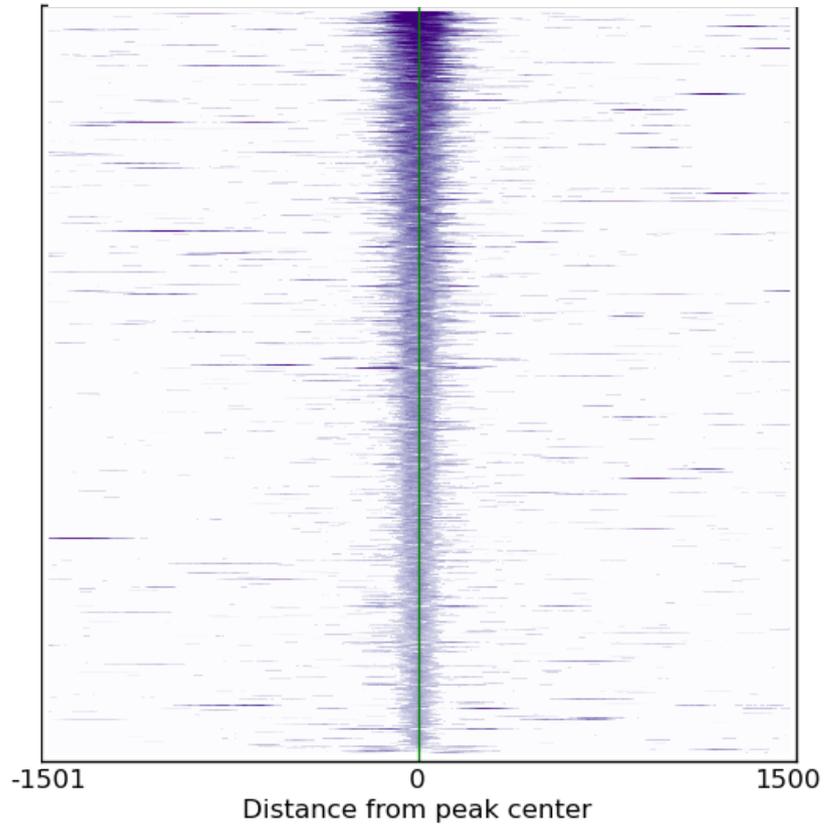
Peak-calling algorithm identifies ~28,000  
CBP binding sites in two replicates



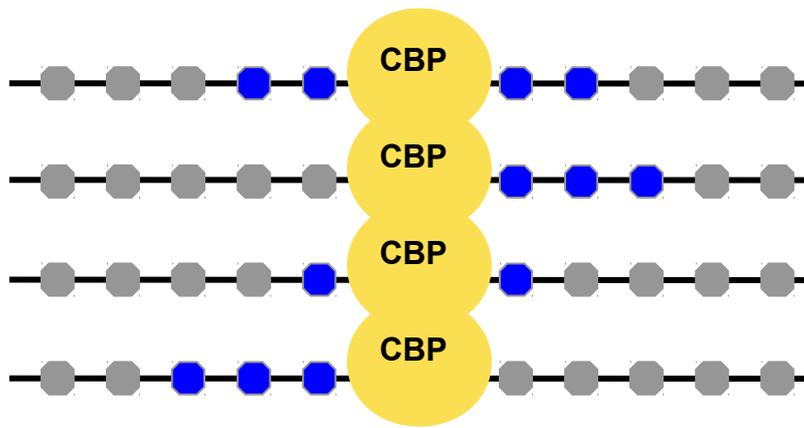
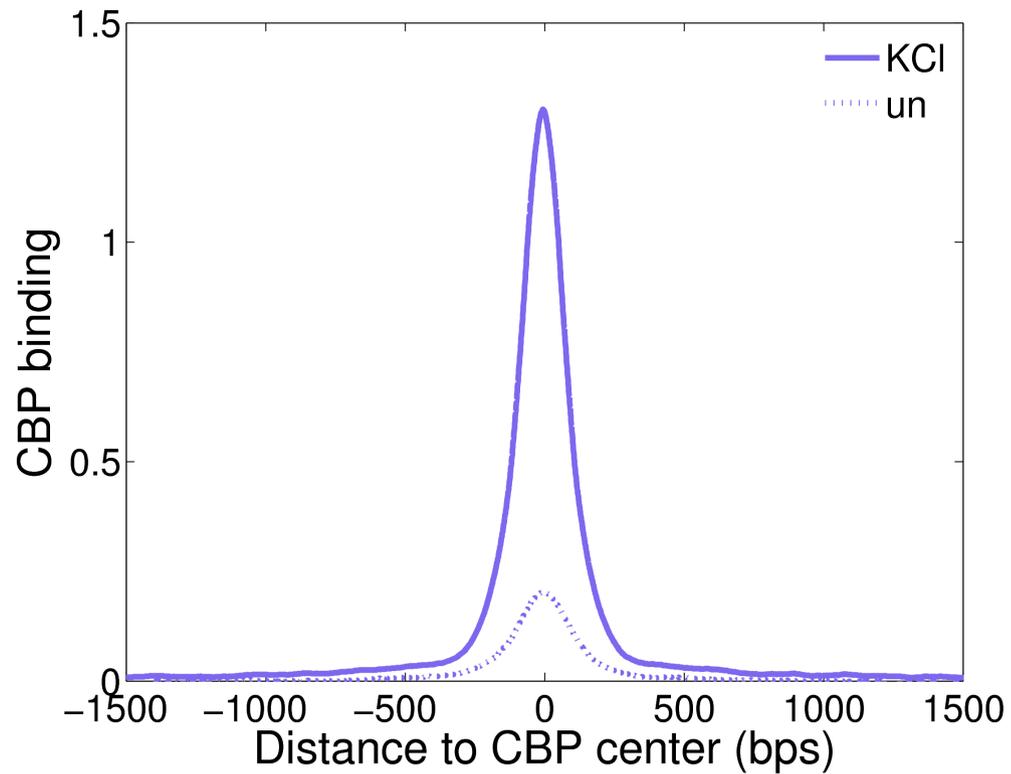
# Align CBP peaks to obtain binding profiles



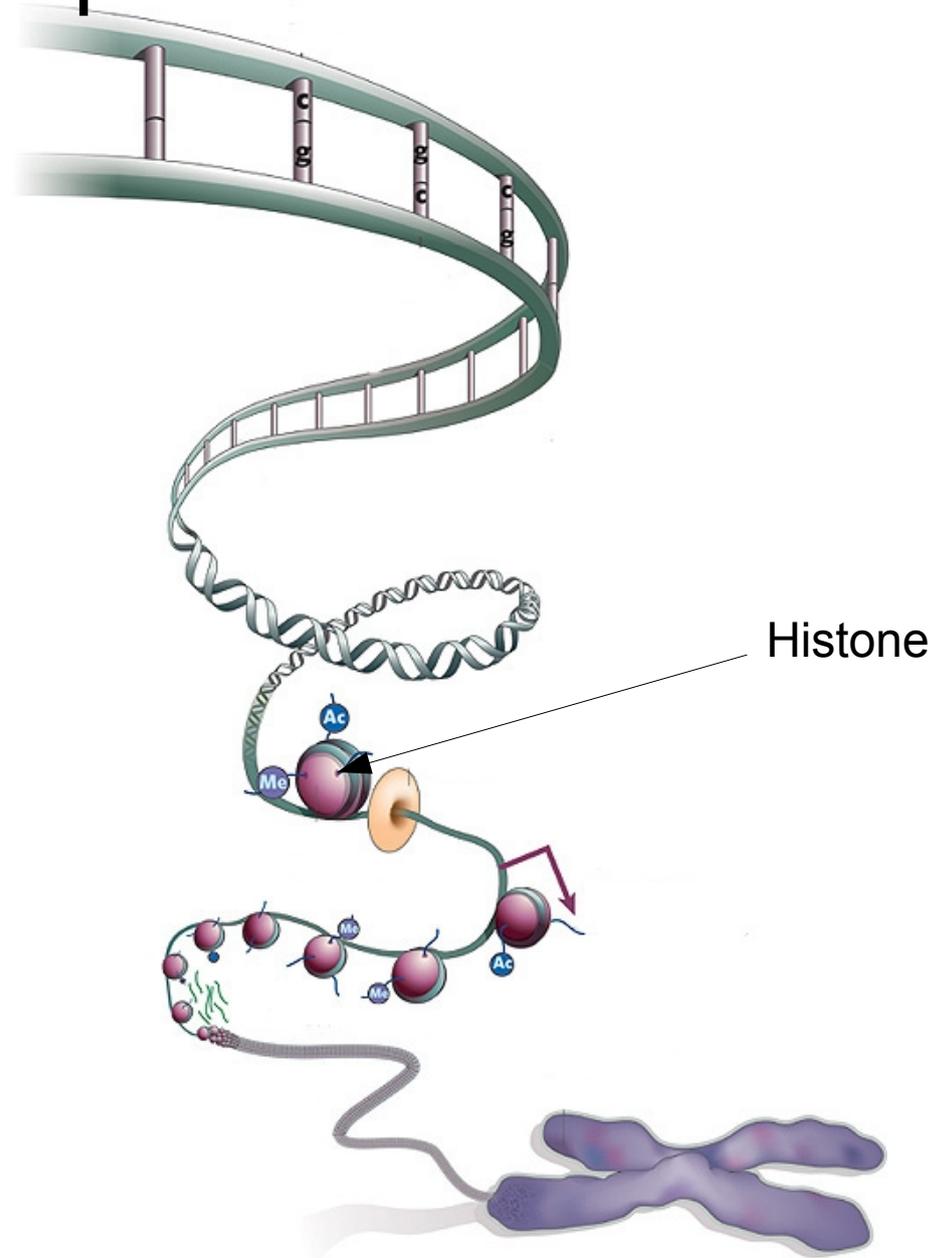
# Align CBP peaks to obtain binding profiles



# Average profile of CBP binding

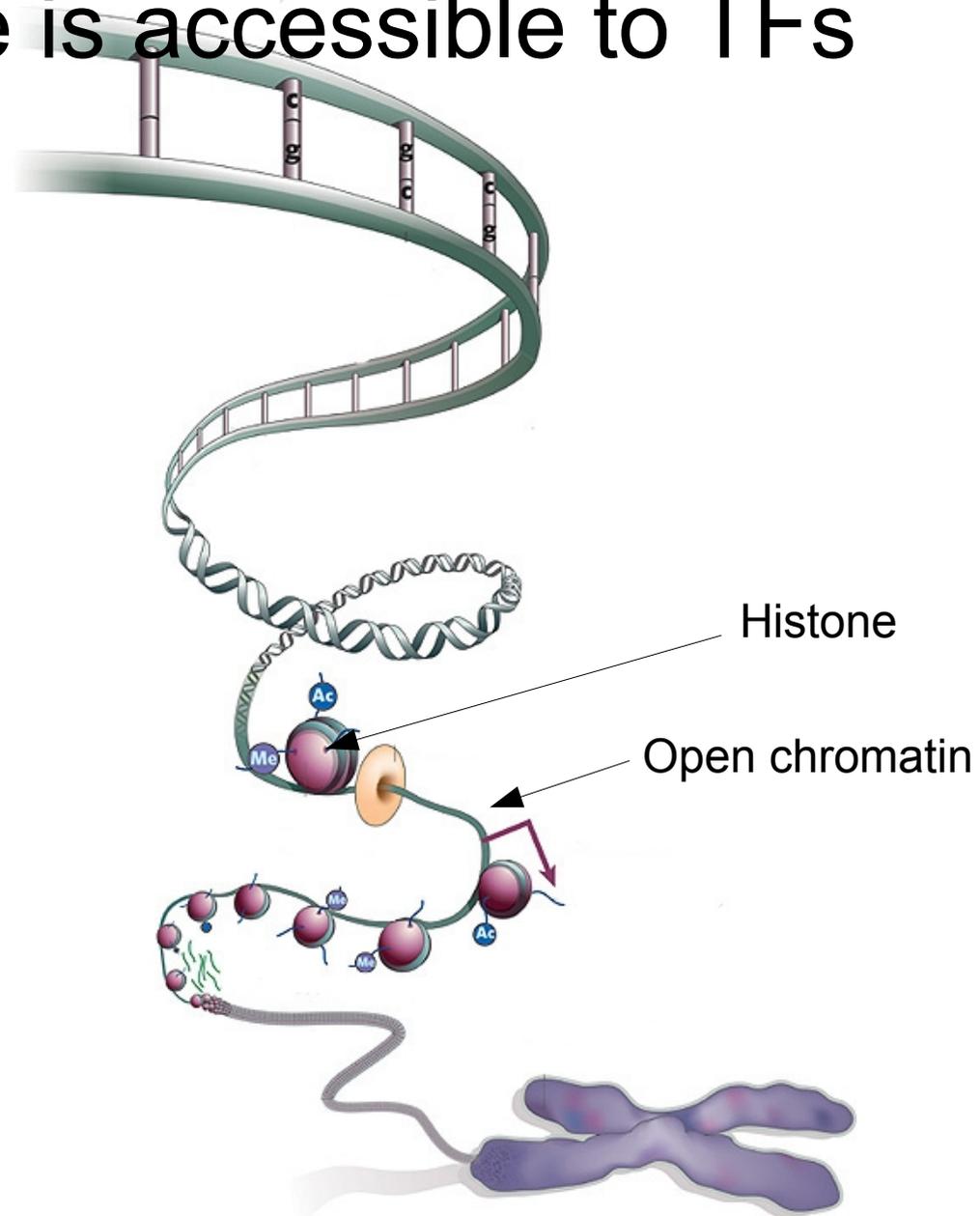


# Histones prevent transcription factors from binding to DNA



(ENCODE, 2007)

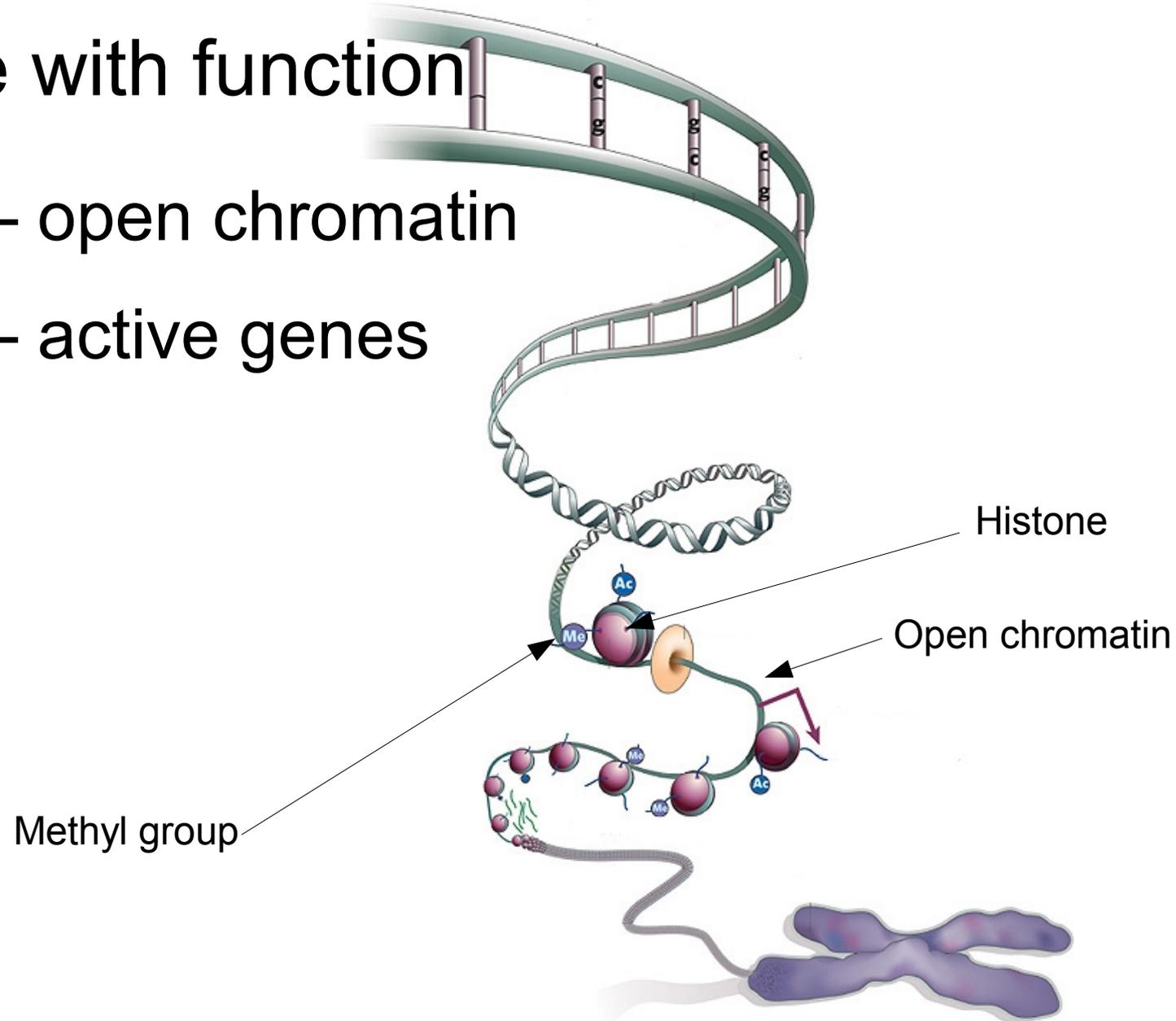
# Only 1% of the genome is accessible to TFs



(ENCODE, 2007)

# Post-translational modifications of histone tails correlate with function

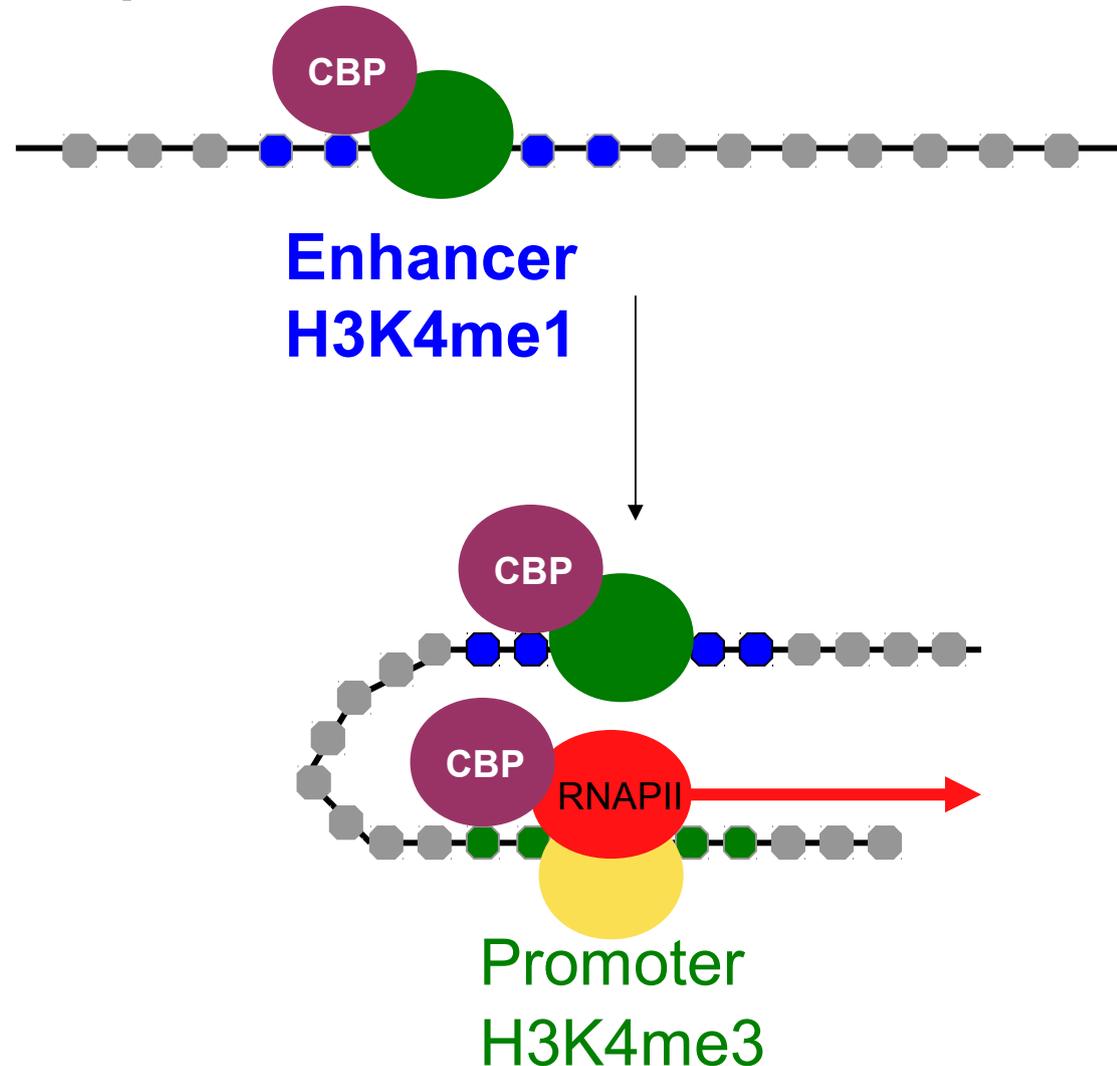
- **H3K4me1** – open chromatin
- **H3K4me3** – active genes



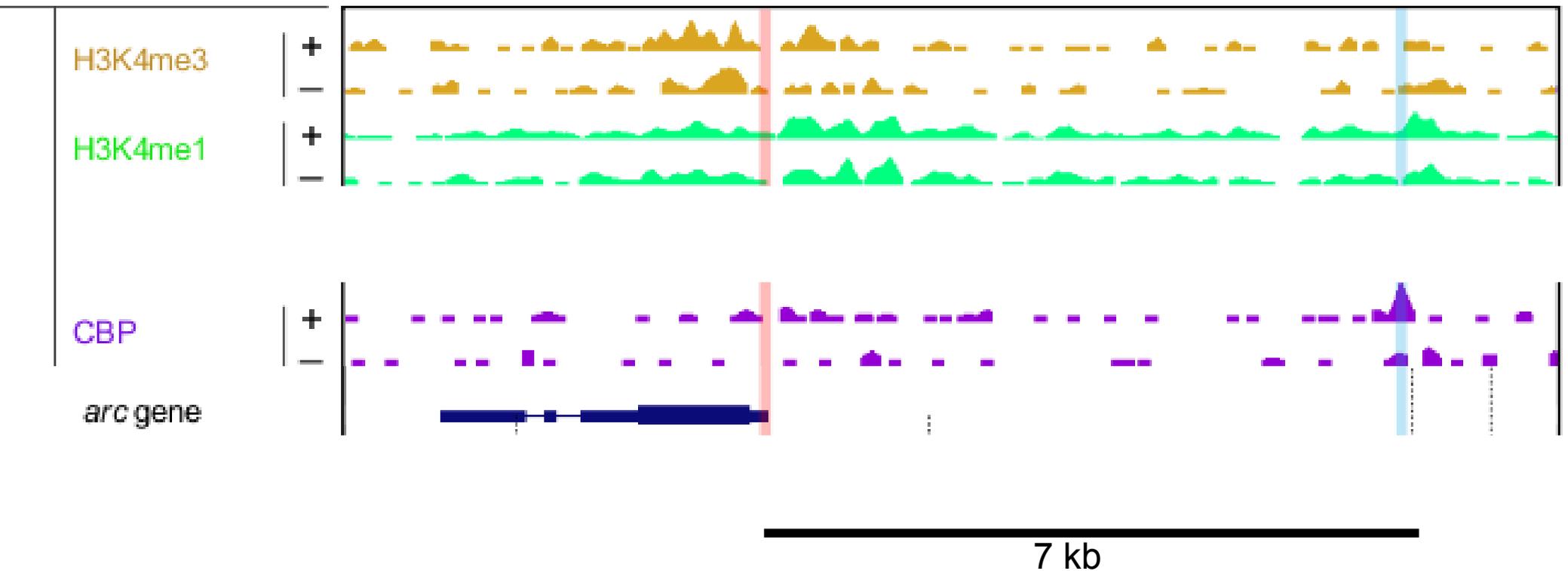
(ENCODE, 2007)

# A combination of CBP and histone modifications identifies putative enhancers

- **CBP** binding
- **H3K4me1** flanking
- **H3K4me3** absent
  - Many unannotated promoters in the genome



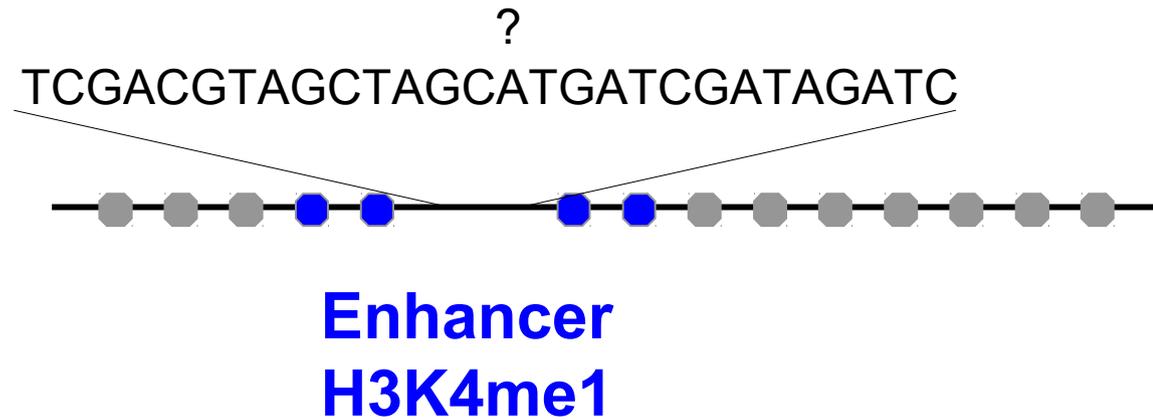
Distal CBP peaks have high levels of H3K4me1 and low levels of H3K4me3



We identified ~12,000 activity-dependent enhancers throughout the genome

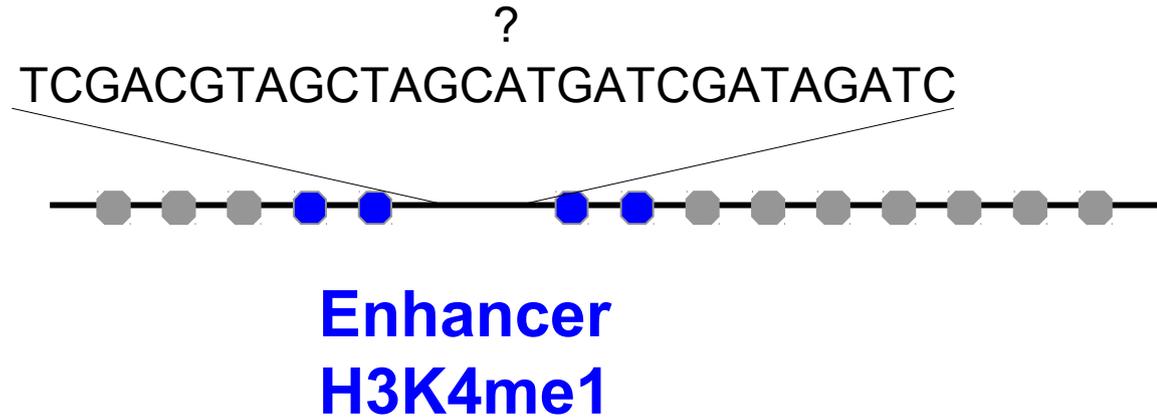
- **CBP** peak
- **High** levels of flanking **H3K4me1**
- **Low** levels of **H3K4me3**

# What TFs bind to enhancers?



- CBP -  
CREB Binding  
Protein  
– >50 partners

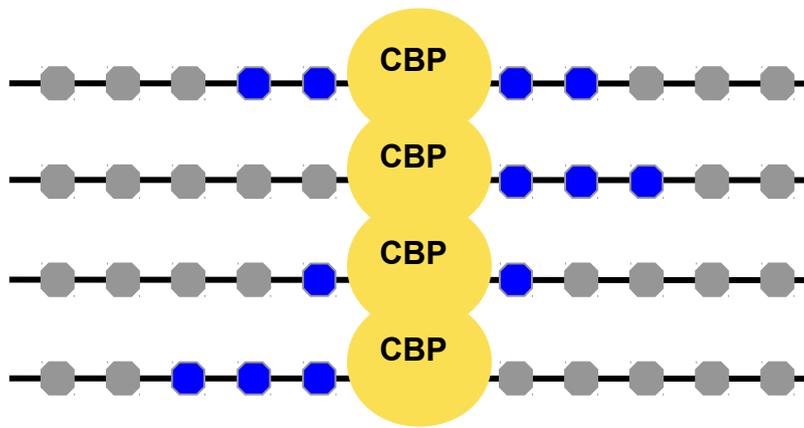
# ~100 enriched motifs at enhancers



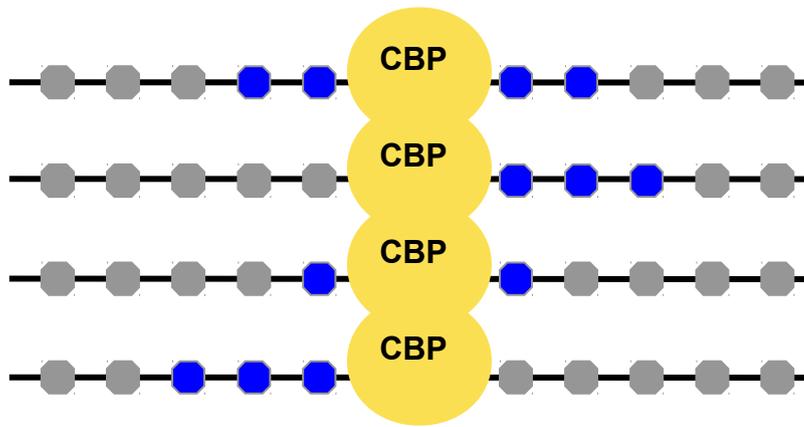
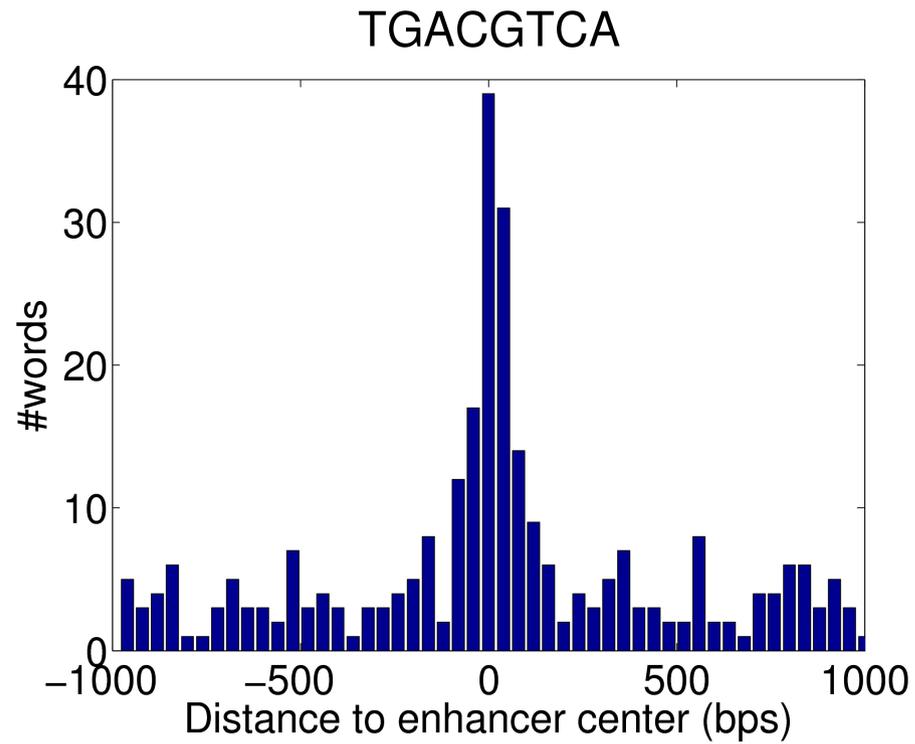
TCAGGCTGATGACGTCAAACCGTCGTTA  
ACCTTTTGACGTCAAATTTACGCTAGTAT •  
TCGACGTAGCTAGCATGATCGATAGATC  
CGTGACGTCA GTGCTCGTAAATCATAAG

• CBP -  
CREB Binding  
Protein

- >50 partners



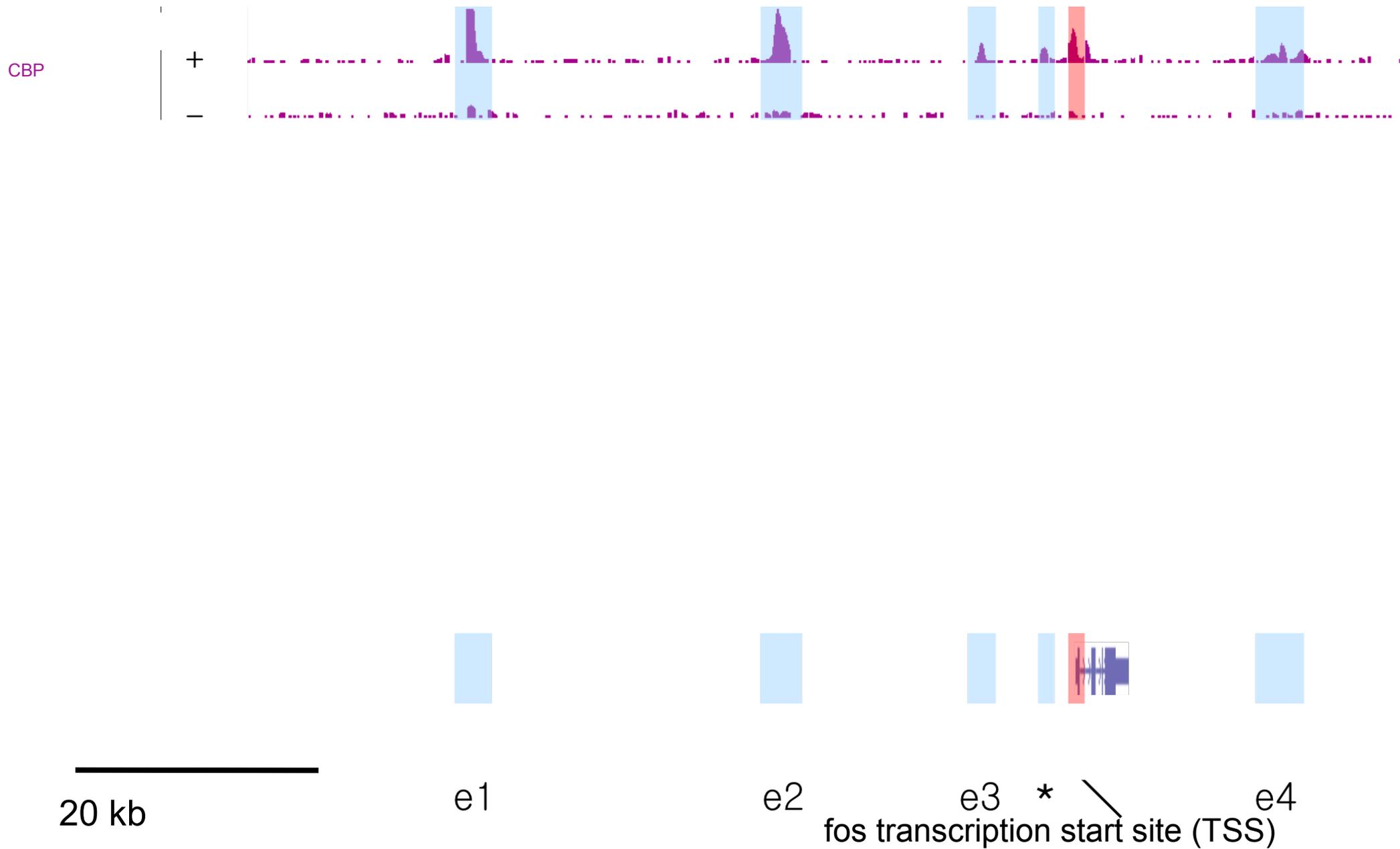
# Enrichment of the CRE motif



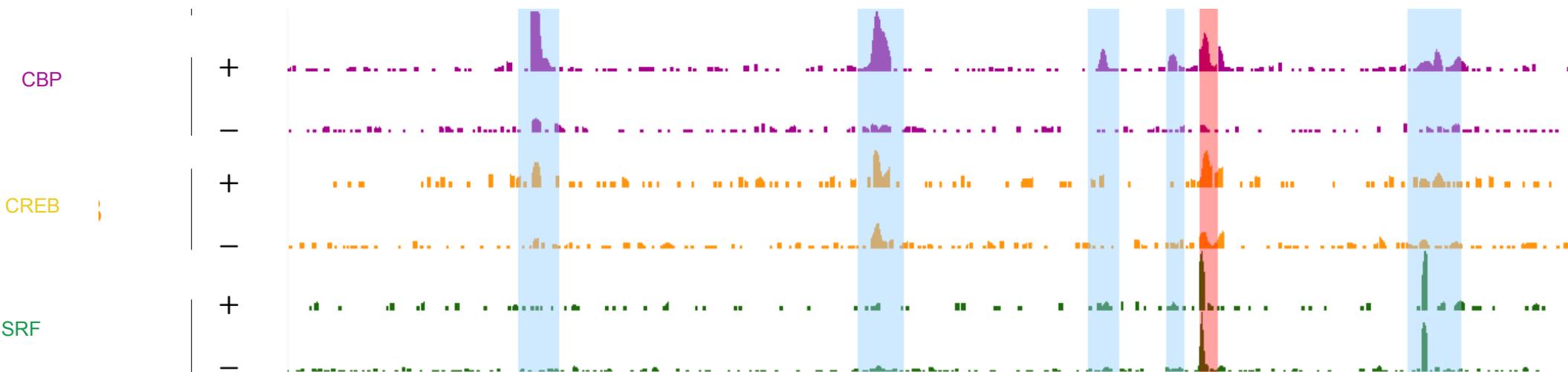
# Motifs for several known TFs were identified

<b>Word</b>	<b>Enrichment</b>	<b>Known TF</b>
TGASTCA	4.74	Fos/Jun
TGACGTCA	6.41	Creb
CTAWWWATA	3.34	Srf
TCGTG	1.56	Npas4
CTGCCAAA	3.34	?

# Several enhancers at the *fos* locus



# SRF and CREB binding at *fos* enhancers



20 kb

e1

e2

e3

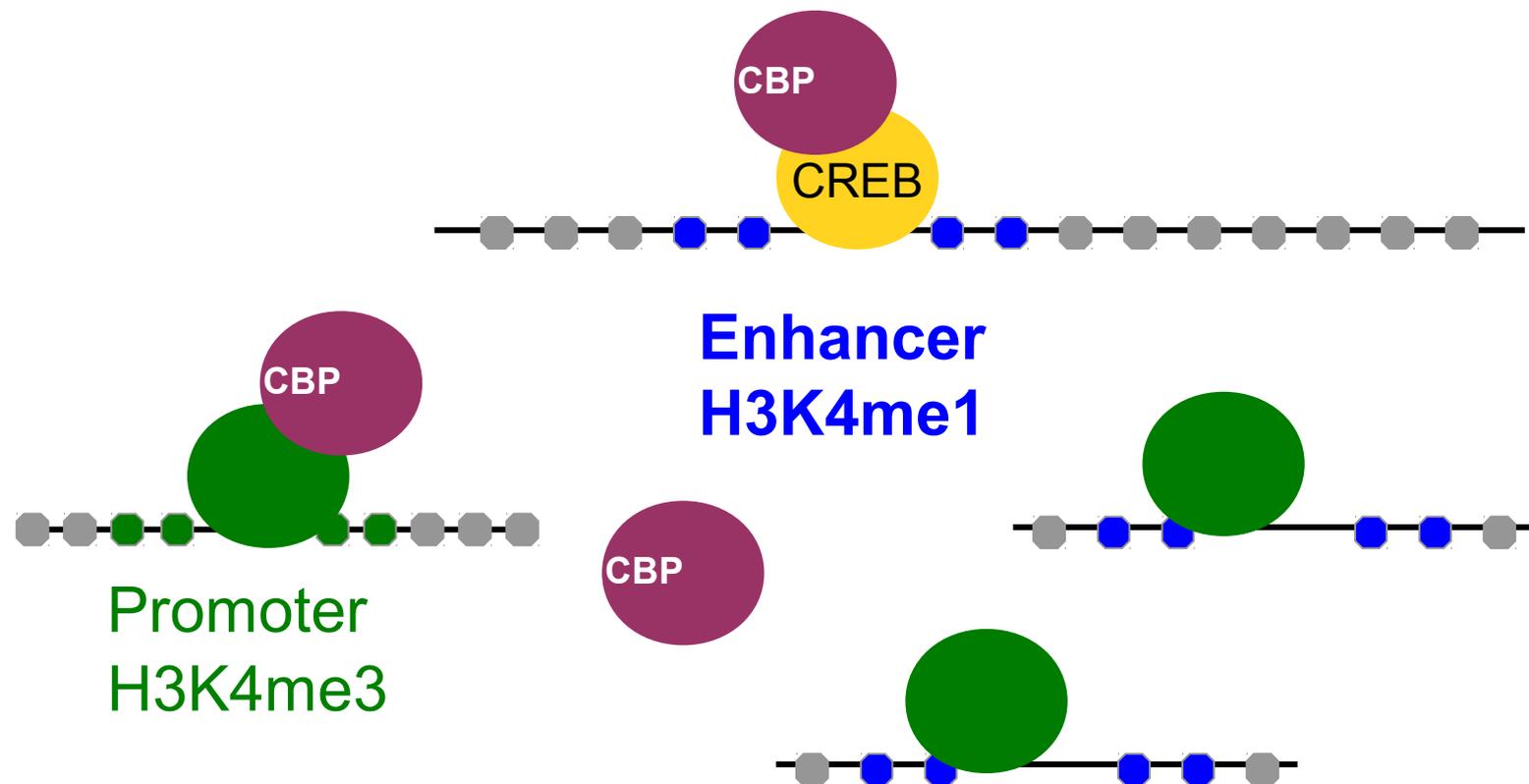
\*

*fos* transcription start site (TSS)

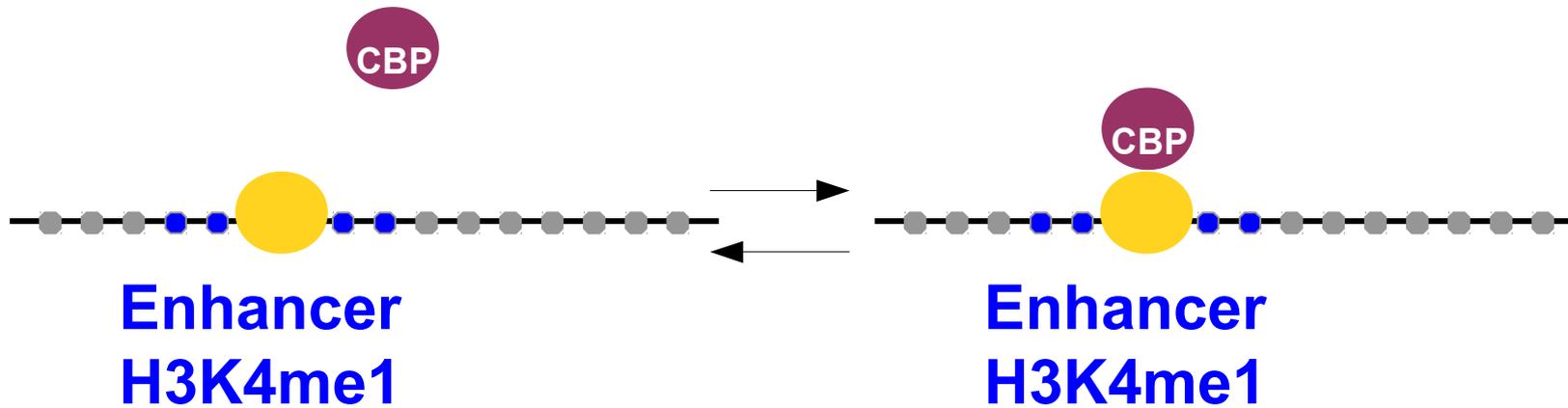
e4

# Is CBP binding determined by a combination of TFs?

- Enriched for ~100 sequence motifs
- ChIP-seq reads predicted by sequence

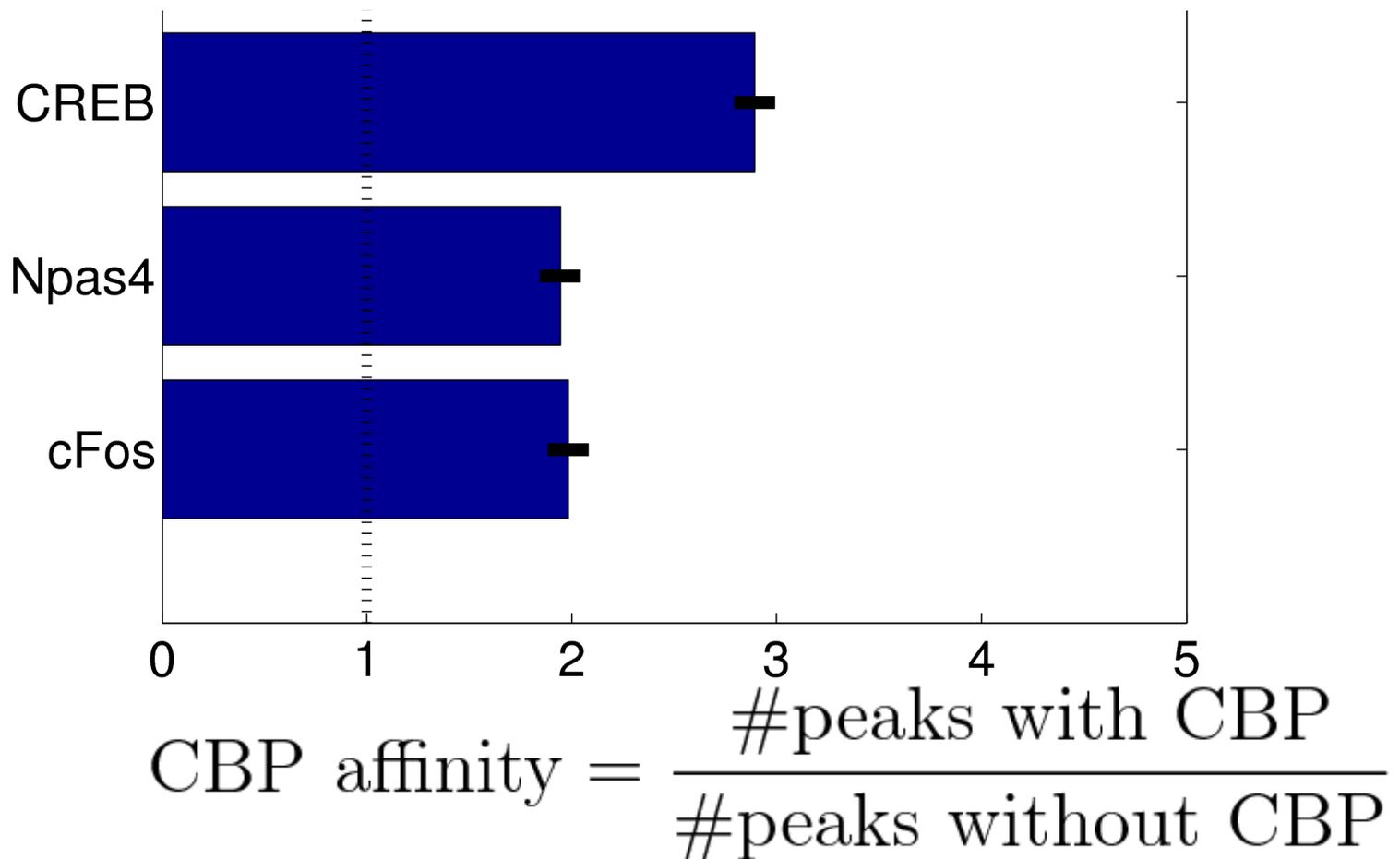


# TFs compete for CBP

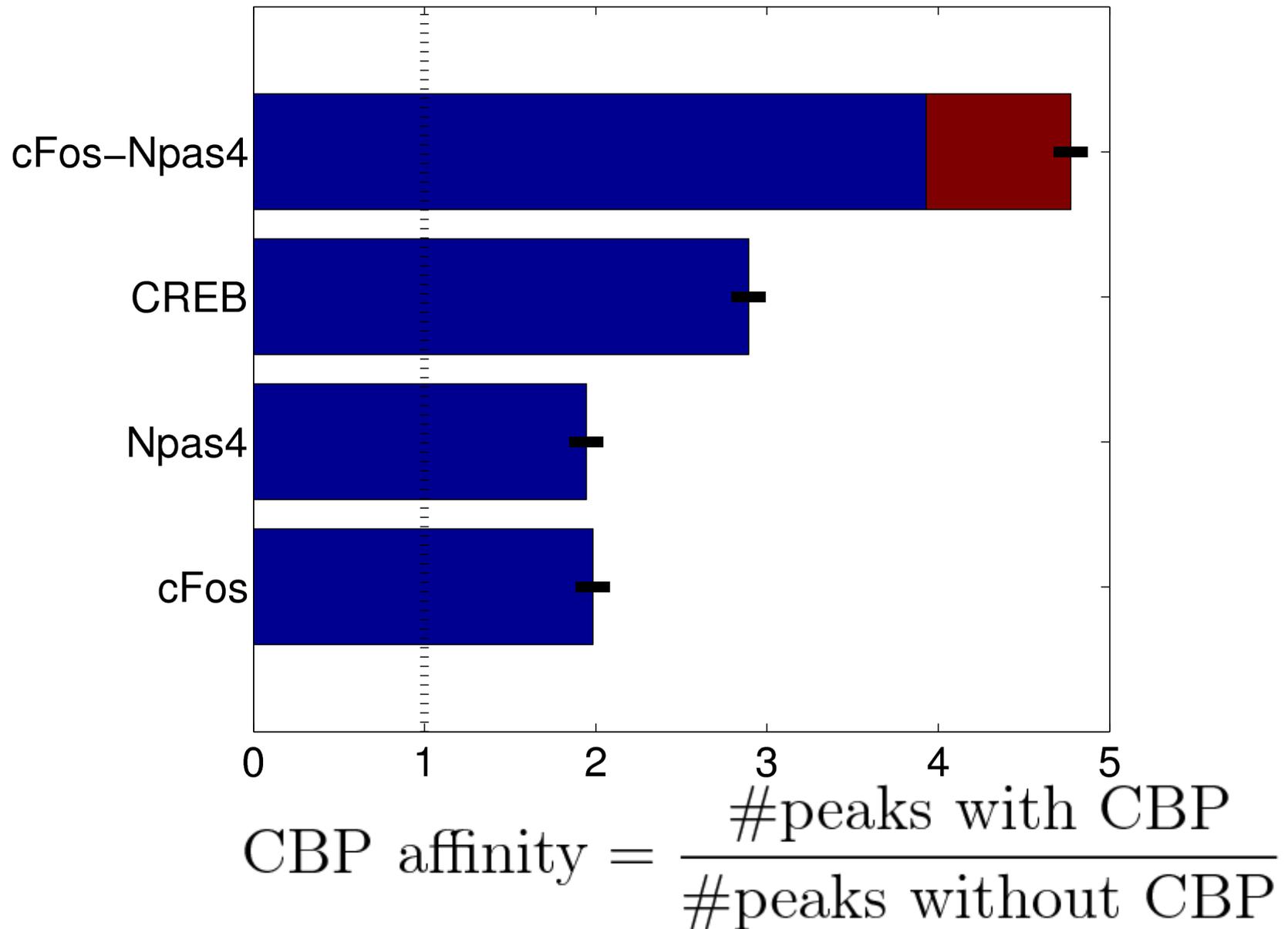


$$\text{CBP affinity} = \frac{\# \text{peaks with CBP}}{\# \text{peaks without CBP}}$$

# Tfs have different affinities for CBP binding

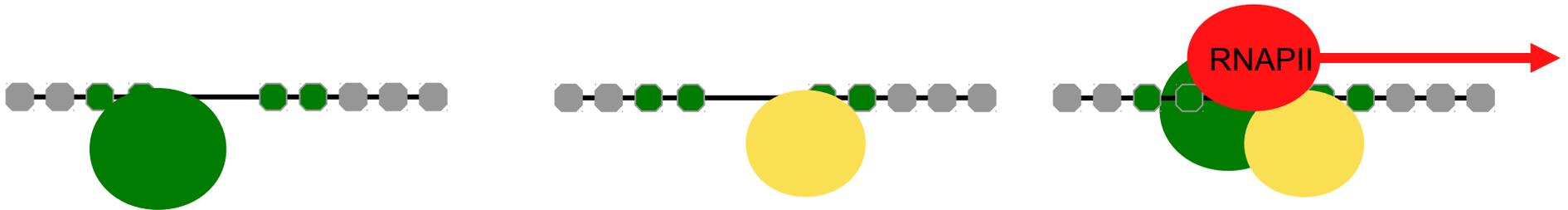


# Synergistic effects for combinations of TFs

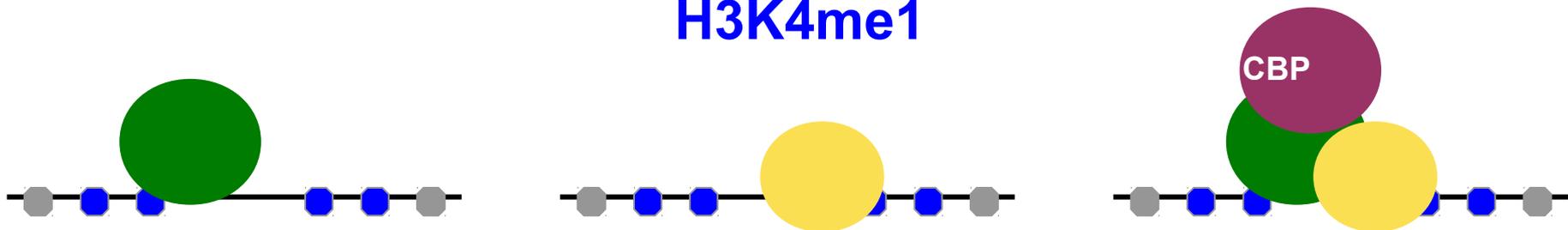


# Combinatorial code determines RNAPII levels at promoters and CBP at enhancers

Promoter  
H3K4me3

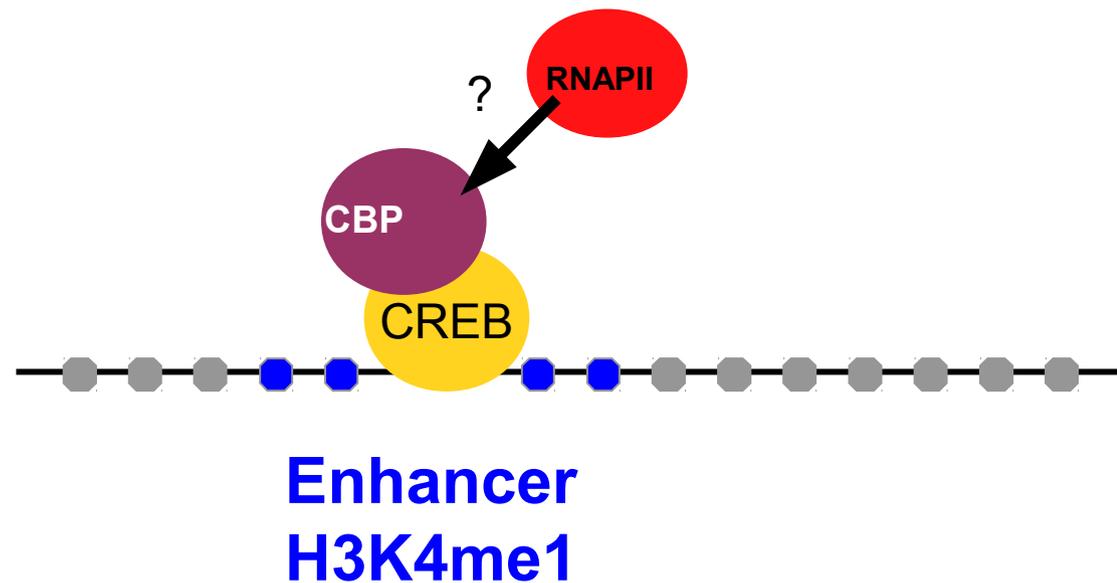


Enhancer  
H3K4me1

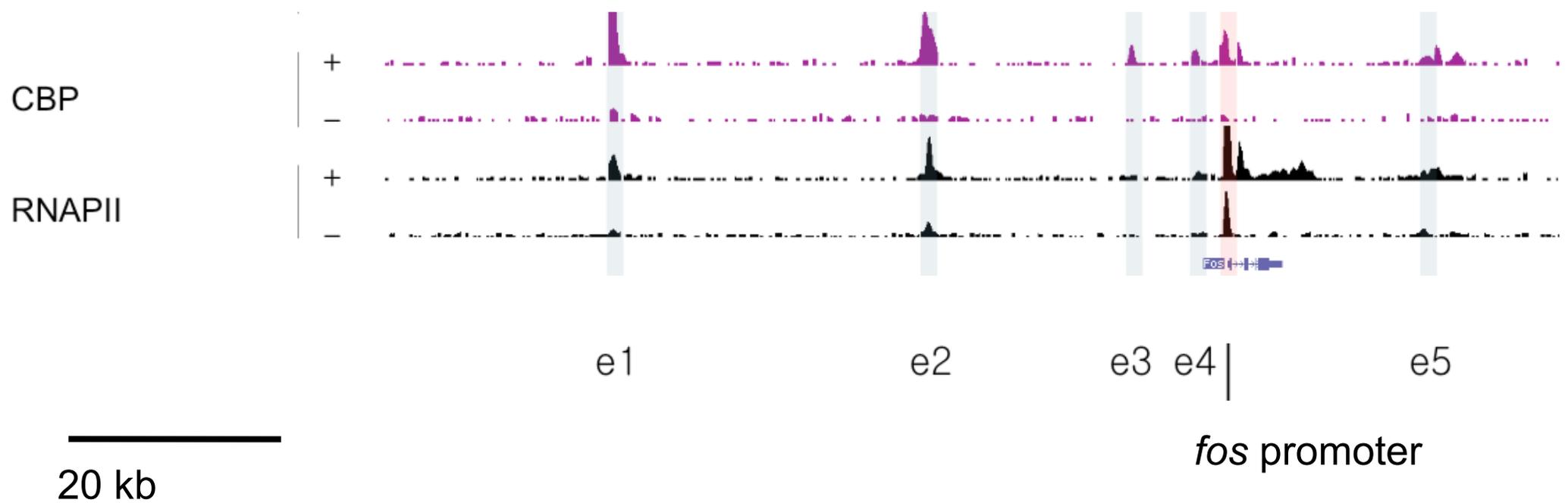


# What is the function of CBP at enhancers?

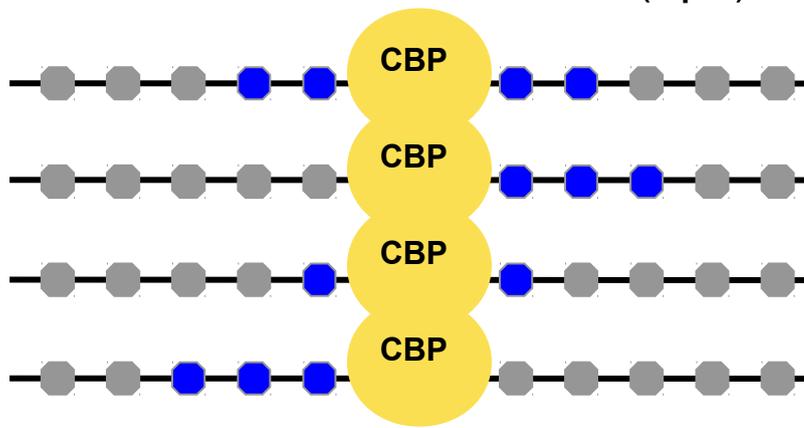
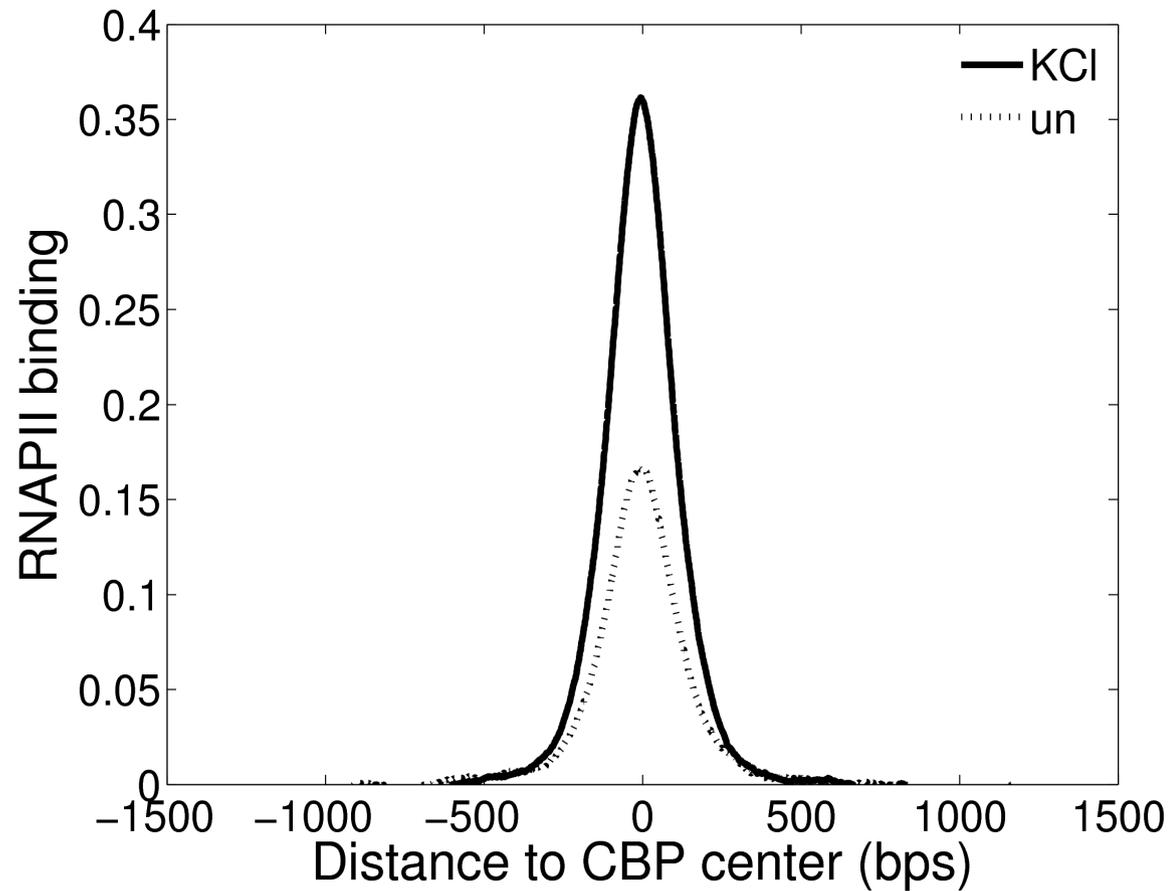
- Enriched for ~100 sequence motifs
- CBP binding determined by other TFs



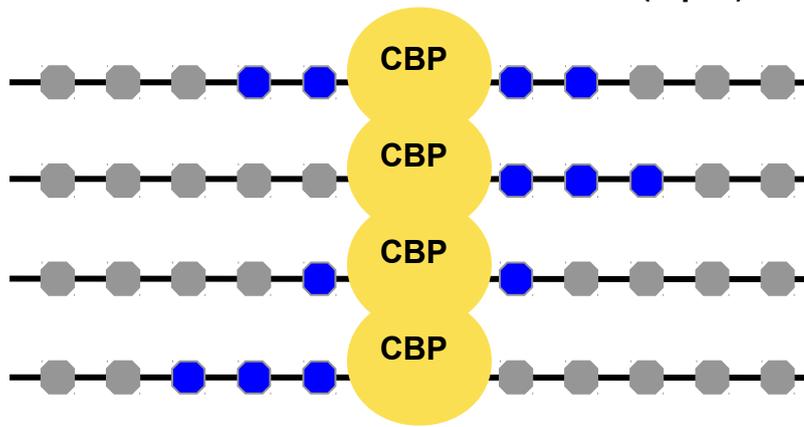
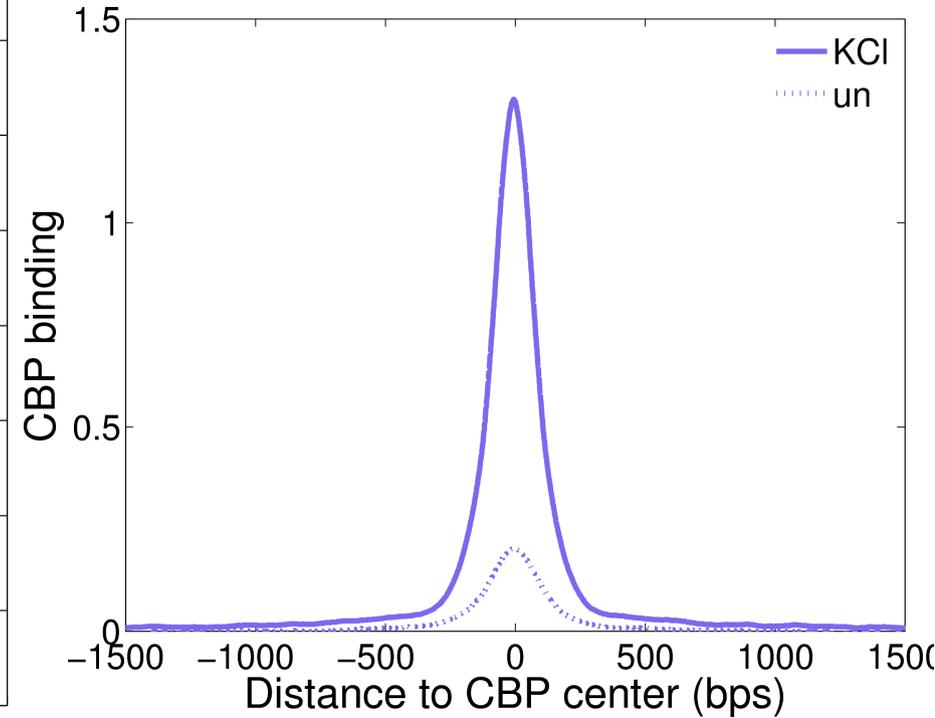
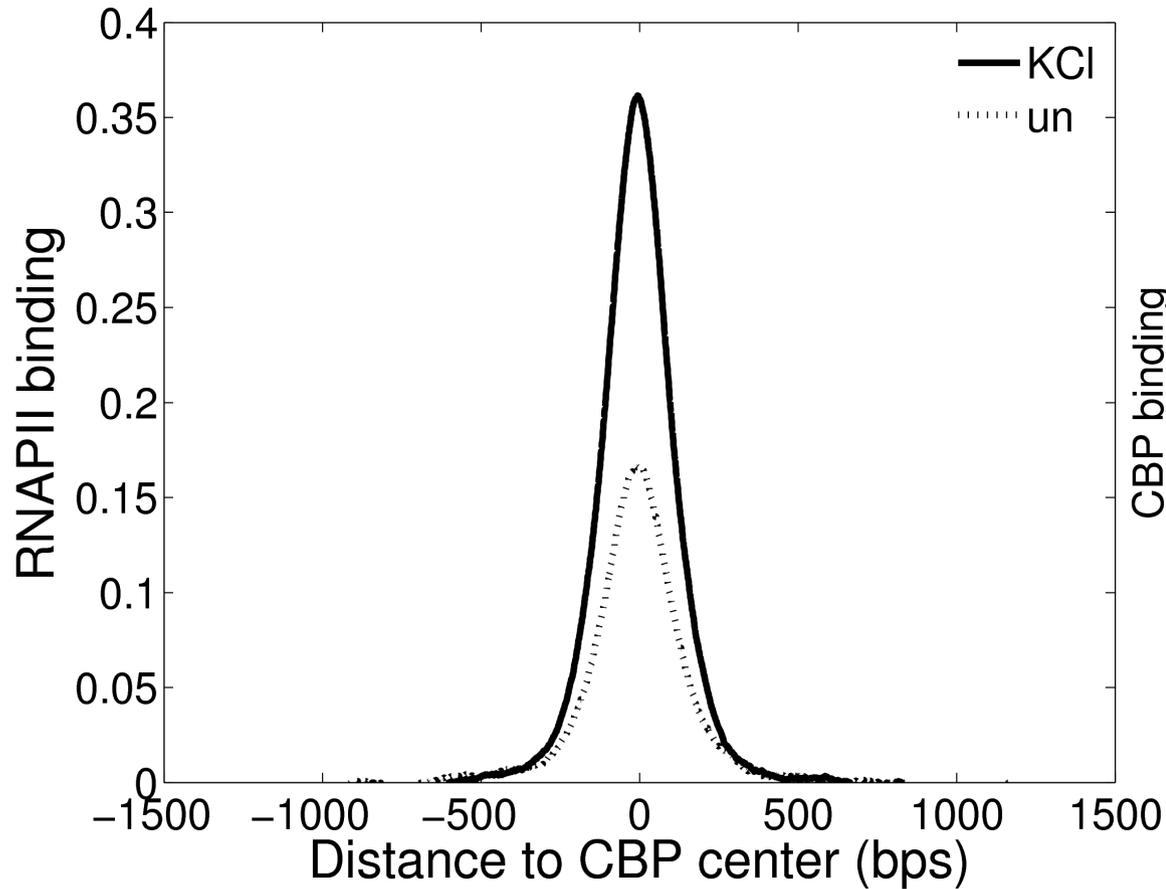
# RNAPII is recruited to CBP binding sites at the *fos* locus



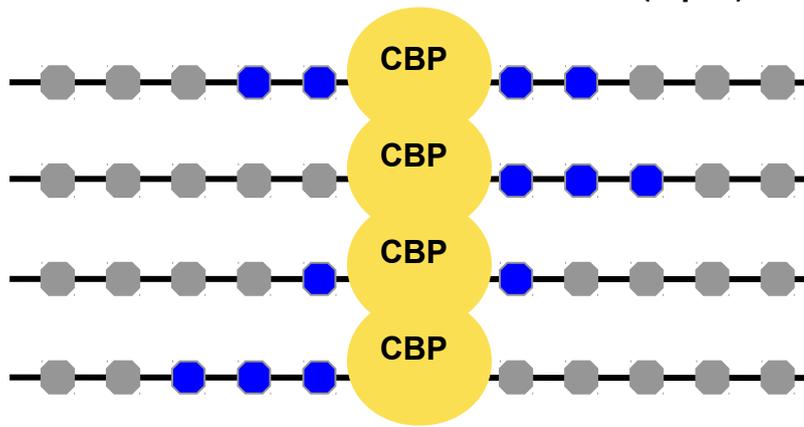
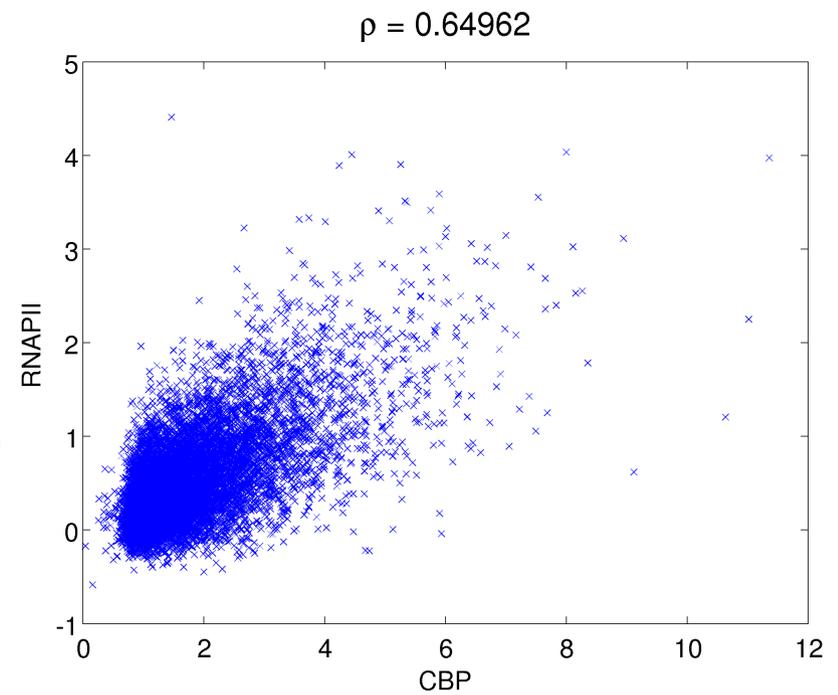
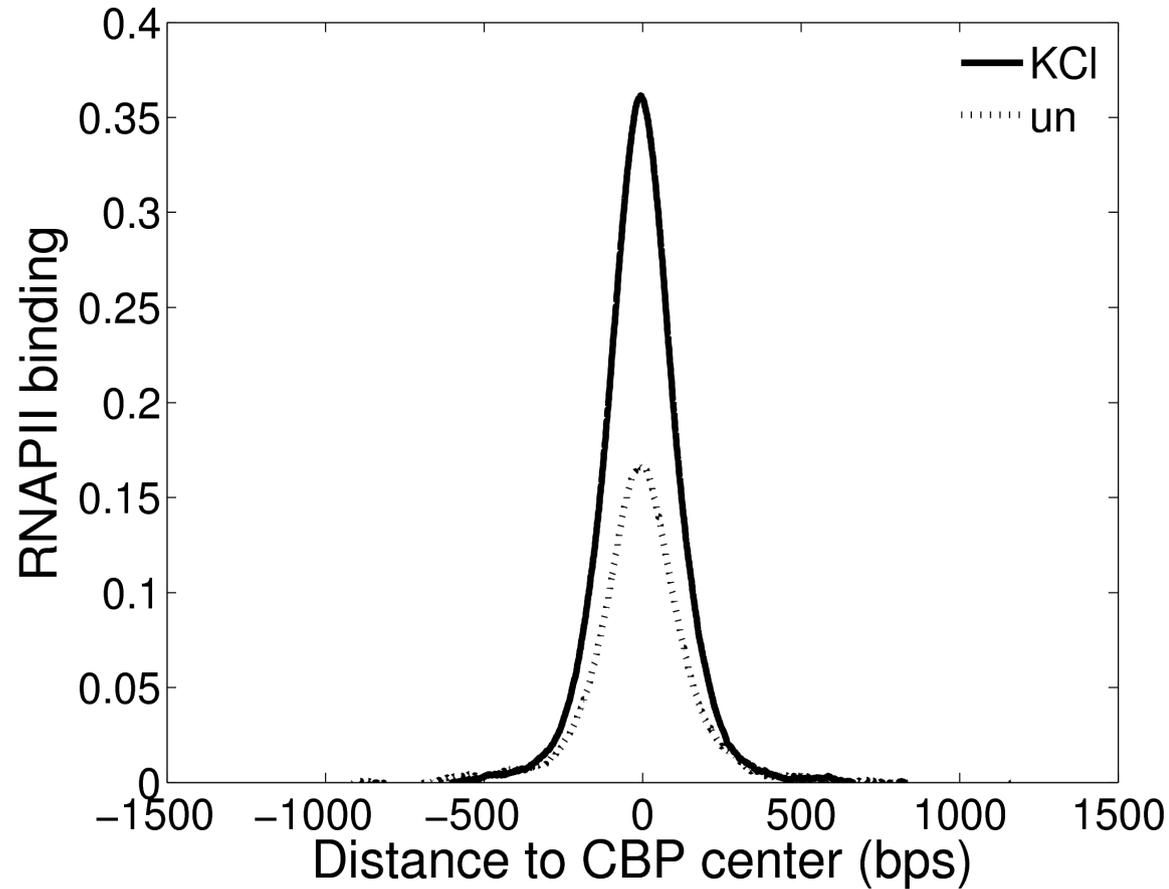
# RNAPII is recruited at enhancers



# RNAPII is recruited at enhancers

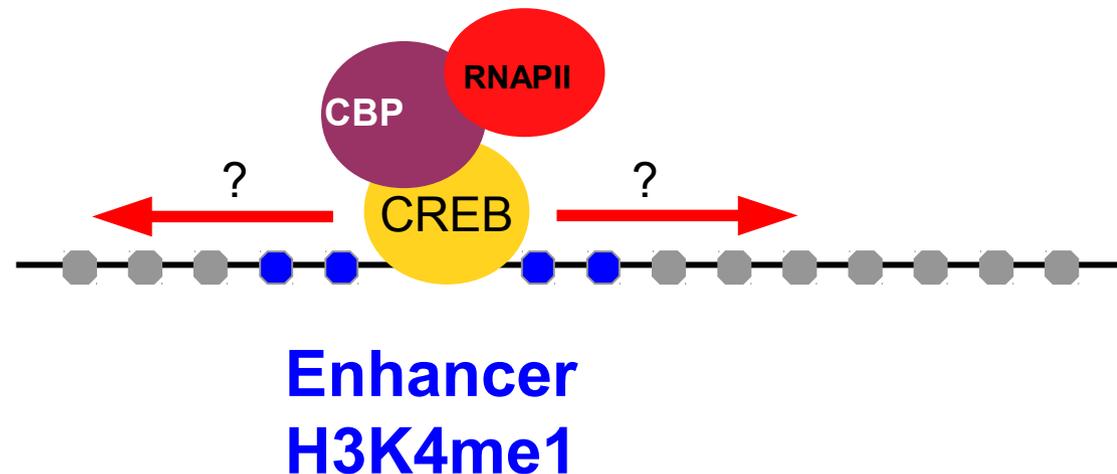


# RNAPII is correlated with CBP



# What is the function of RNAPII at enhancers?

- Enriched for ~100 sequence motifs
- ChIP-seq reads predicted by sequence
- CBP binding determined by other TFs
- CBP recruits RNAPII

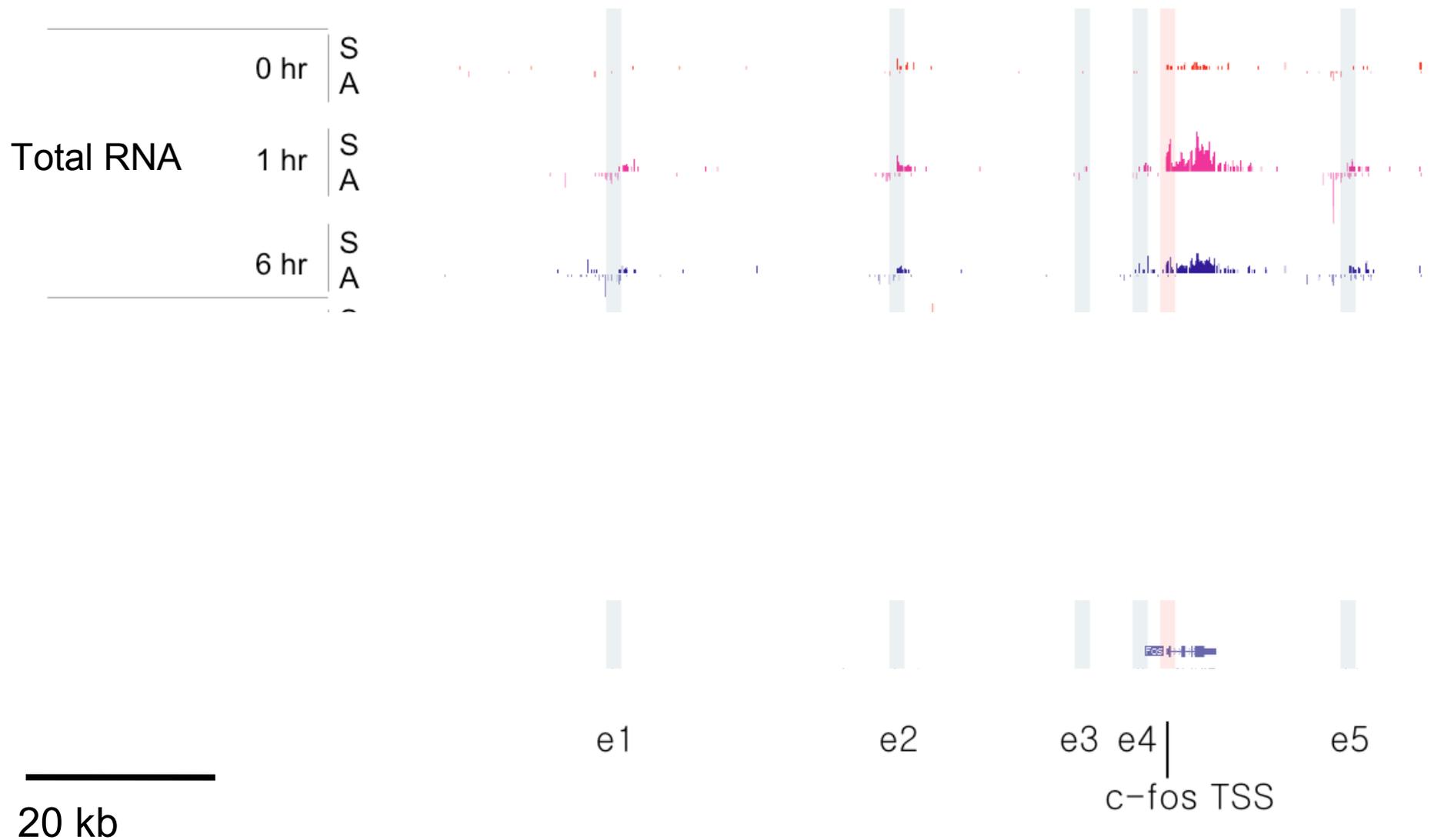


# RNA-Seq finds transcribed parts of the genome

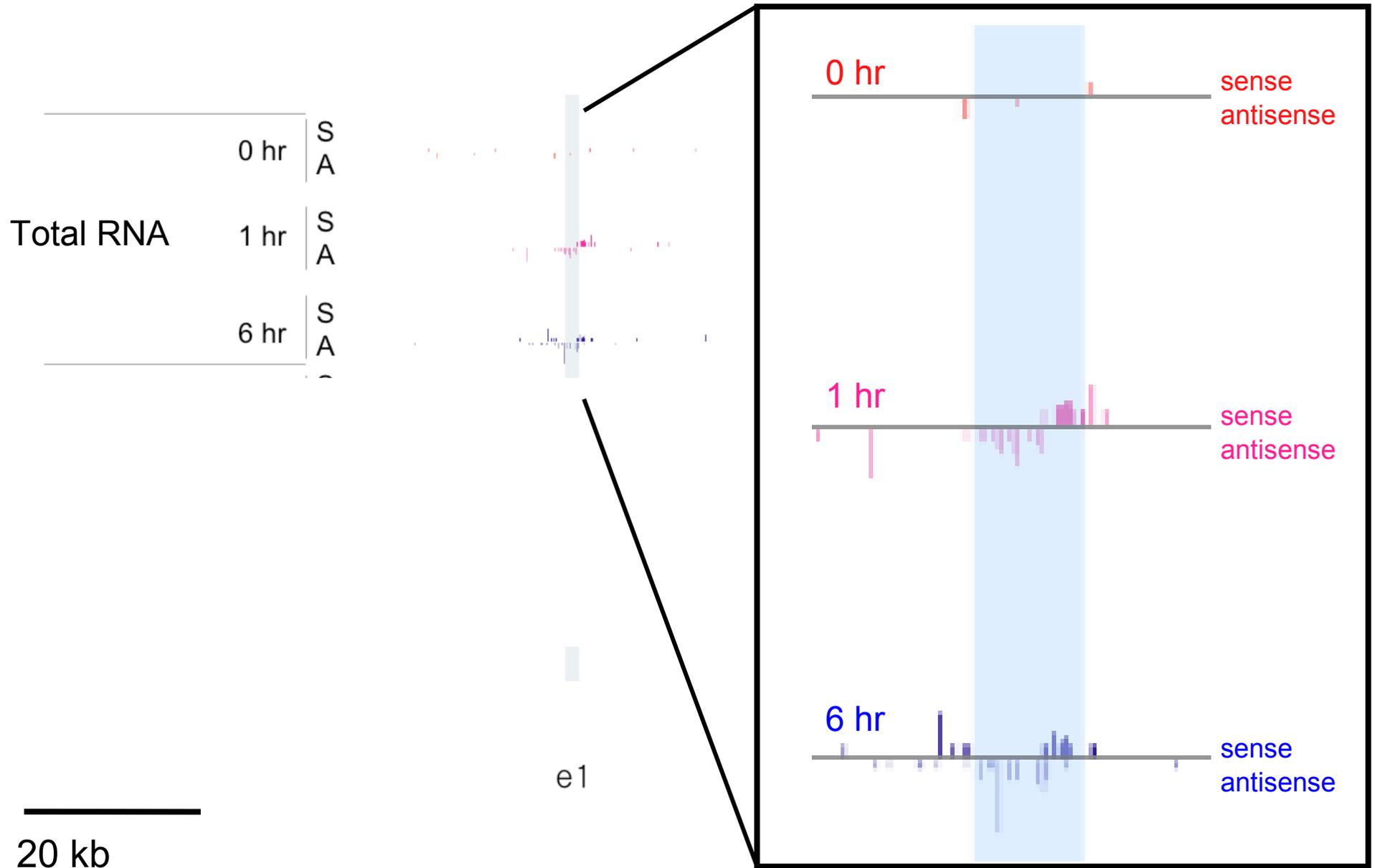
- Short **reads** mapped to reference genome
- $\sim 5 \times 10^6$  reads
- #reads  $\sim$  RNA



# Transcription of total RNA at the *fos* locus

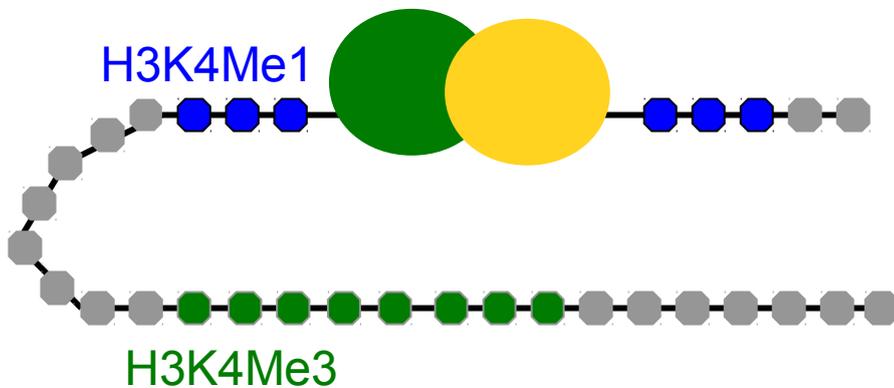


# Transcription at enhancers is activity-dependent

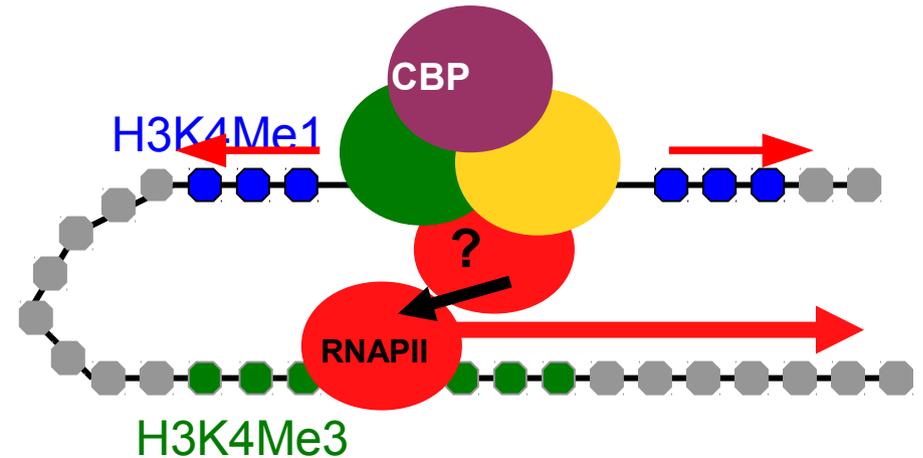


# Enhancer RNAs (eRNAs) novel species

Before neuronal activation

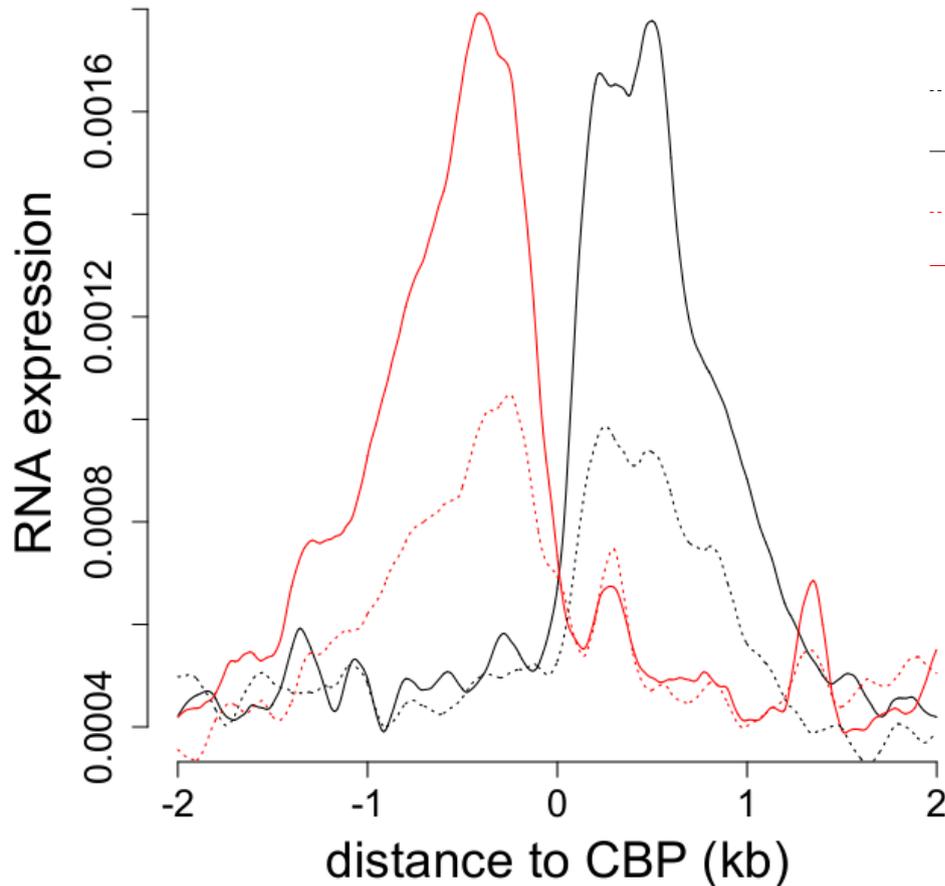


After neuronal activation

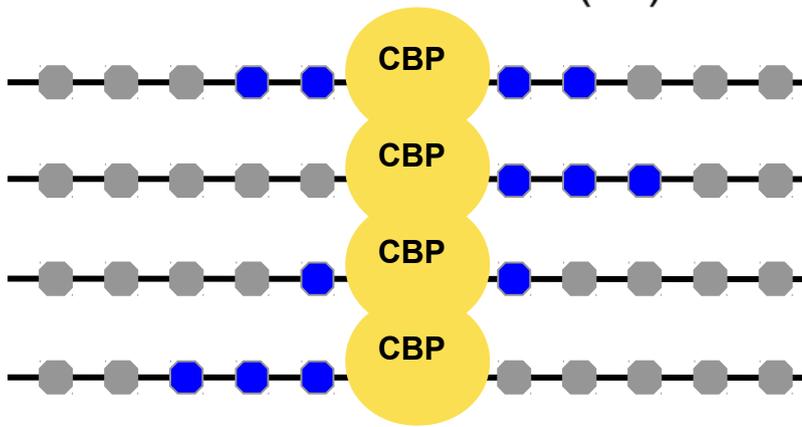


- mRNA, rRNA, tRNA, miRNA, snRNA, snoRNA, siRNA, piRNA, lncRNA, ... ?

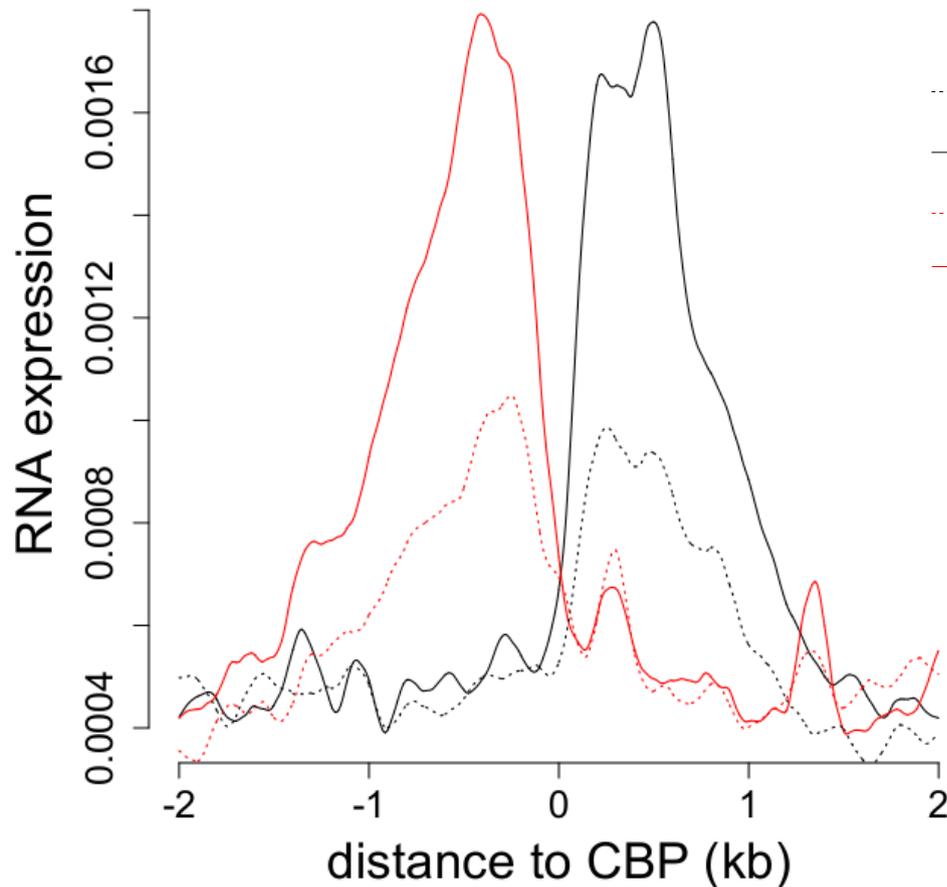
# eRNAs are induced by activity



- Inducible, 2-fold
- ~1 kb
- Bidirectional

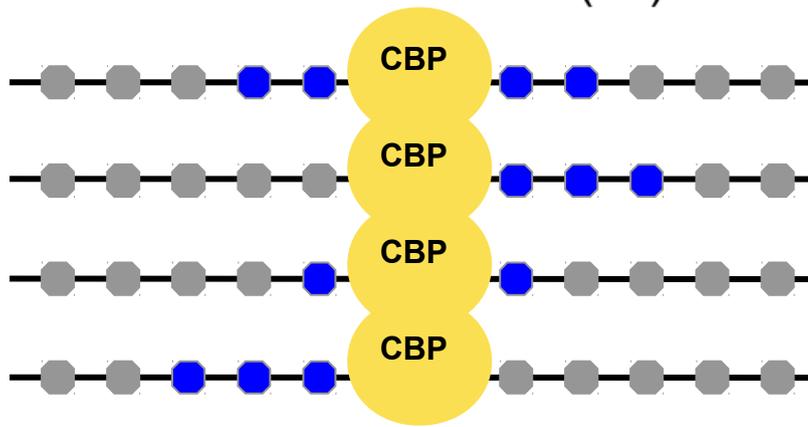


# eRNAs are 100-fold lower than mRNAs



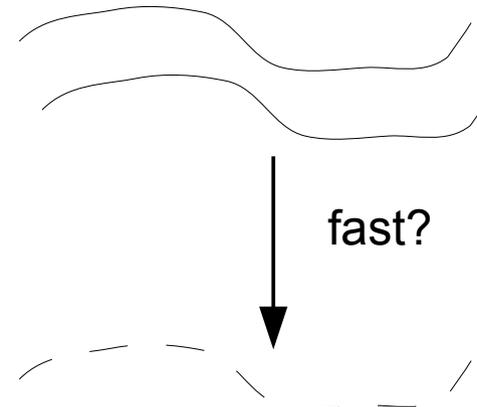
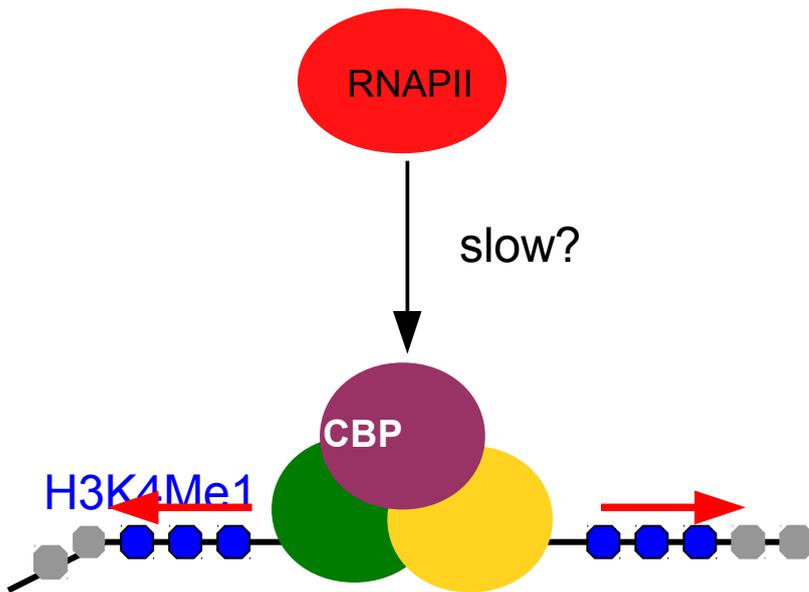
..... Forward strand, un  
—— Forward strand, Kcl  
..... Reverse strand, un  
—— Reverse strand, Kcl

- Inducible, 2-fold
- ~1 kb
- Bidirectional
- 1 in 10k reads eRNA
  - Not at all CBP peaks
- Not protein-coding



# Why do eRNAs have such low abundance?

- eRNA production much slower than mRNA
- eRNA decay much faster than mRNA



# Half life of eRNAs relative to mRNAs

- eRNA production much slower than mRNA
- eRNA decay much faster than mRNA

Length of transcript



$$\frac{\tau_E}{\tau_M} = \frac{E^*}{M^*} \frac{L_E}{L_M} \frac{P_M}{P_E}$$

RNAPII level



half-life



Transcript level

eRNAs half life is less than half an hour

- eRNA production much slower than mRNA
- eRNA decay much faster than mRNA

$$\frac{\tau_E}{\tau_M} = \frac{E^*}{M^*} \frac{L_E}{L_M} \frac{P_M}{P_E}$$

$$\tau_E \sim 10^{-2} \times 1 \times 2 \times \tau_M \sim 2 \times 10^{-2} \times 600\text{min} = 12\text{min}$$

# Estimate consistent with experiments

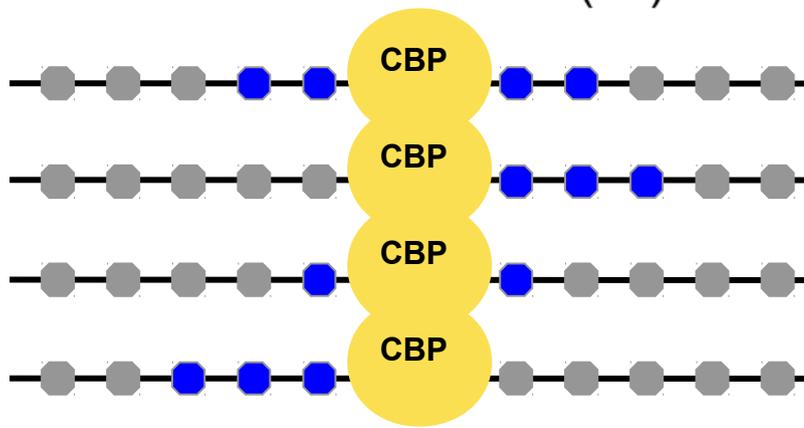
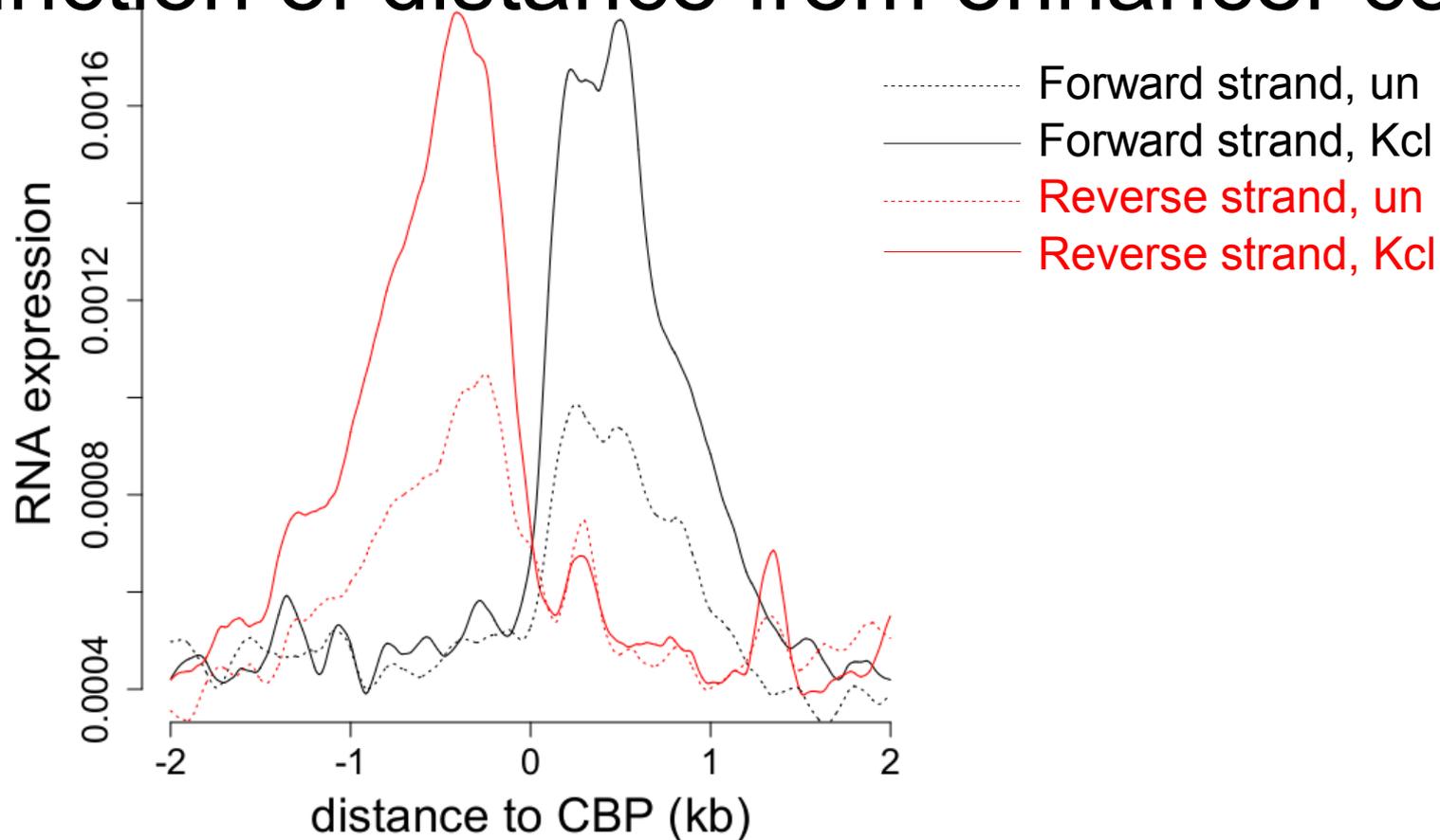
- ~~eRNA production much slower than mRNA~~
- eRNA decay much faster than mRNA

Finally we measured the stability of these transcripts using an actinomycinD chase. In comparison to both the mRNAs generated by the associated protein-coding genes and some known lncRNAs (like Xist and Neat), the upstream non-coding transcripts were very unstable, being reduced by 80% to 90% after a 30 min actinomycinD treatment (indicating a half-life lower than 7.5 min) (Figure 3D and Figure S3). High instability of a subset of lncRNAs both in yeast and mammals mainly depends on degradation by the nuclear exosome [39,40] and often results in the generation of more stable short RNA products [41], which in principle might be responsible for downstream functional effects.

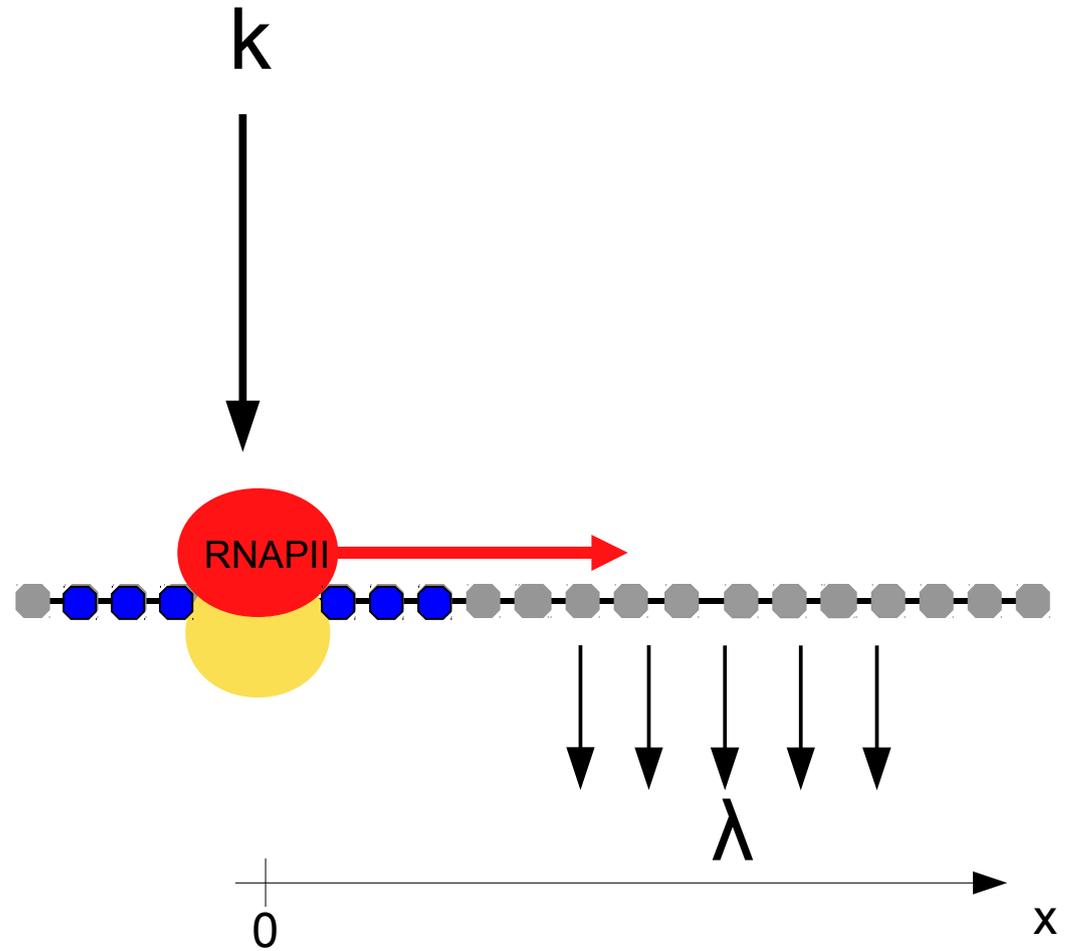
$$\frac{\tau_E}{\tau_M} = \frac{E^*}{M^*} \frac{L_E}{L_M} \frac{P_M}{P_E}$$

$$\tau_E \sim 10^{-2} \times 1 \times 2 \times \tau_M \sim 2 \times 10^{-2} \times 600\text{min} = 12\text{min}$$

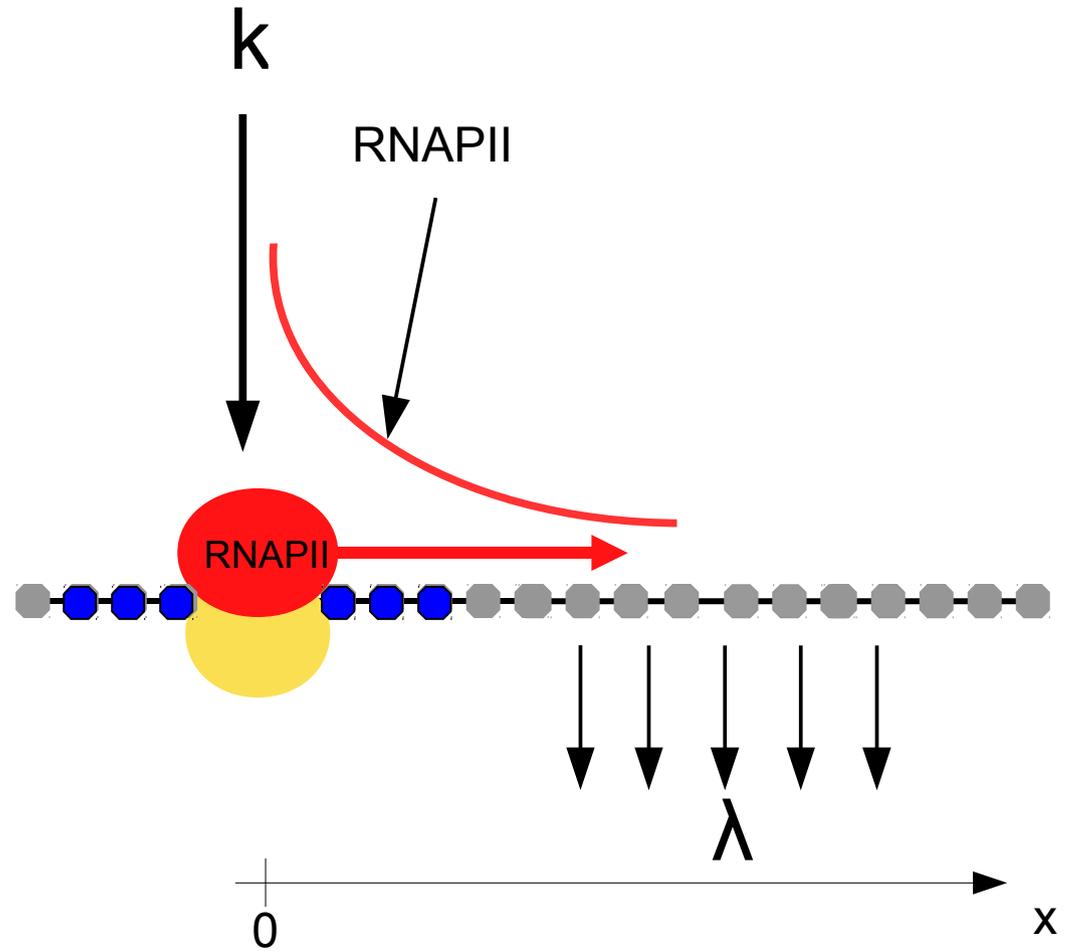
# A quantitative model of eRNA levels as a function of distance from enhancer center



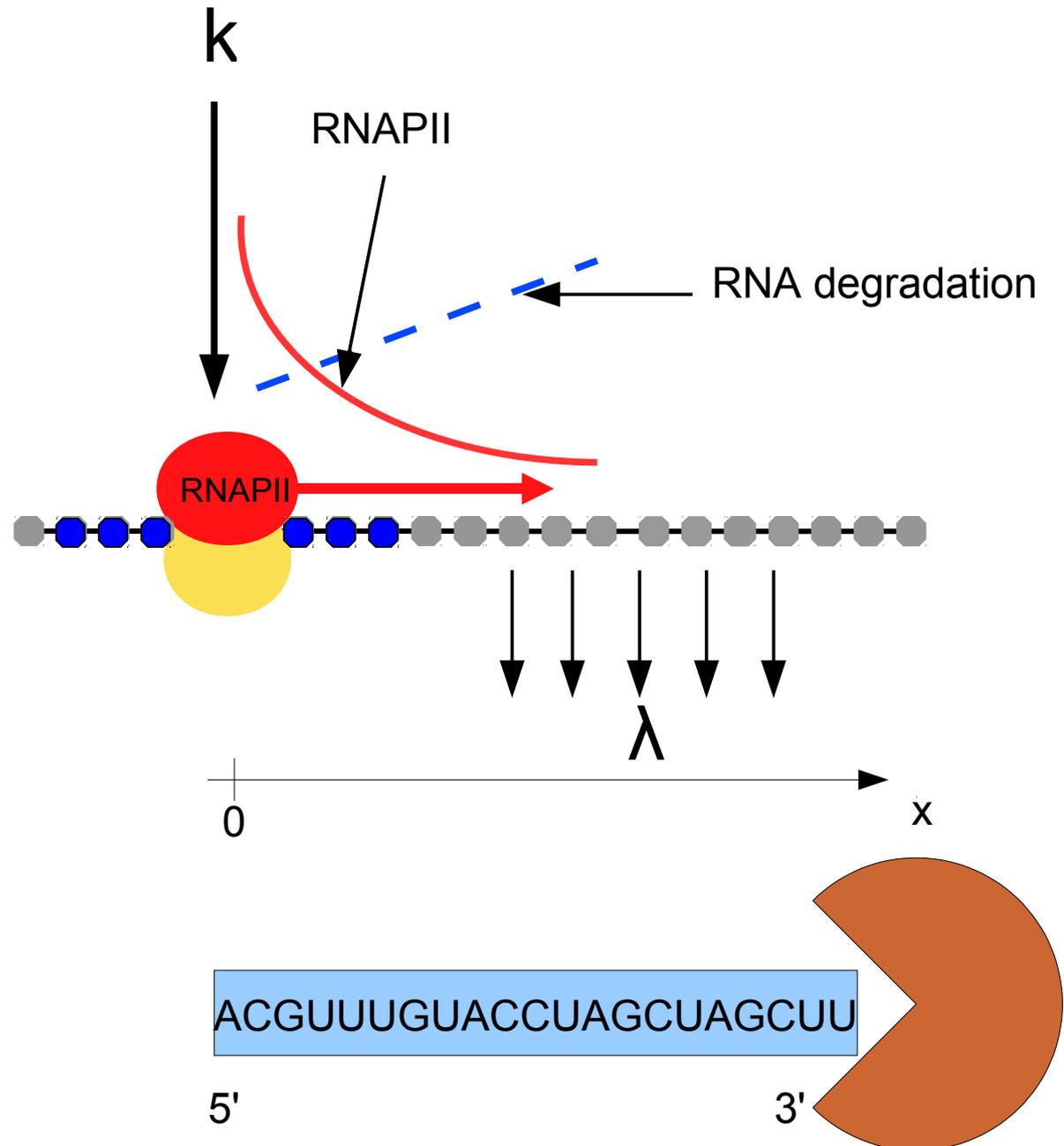
RNAPII binds and falls off at a constant rate



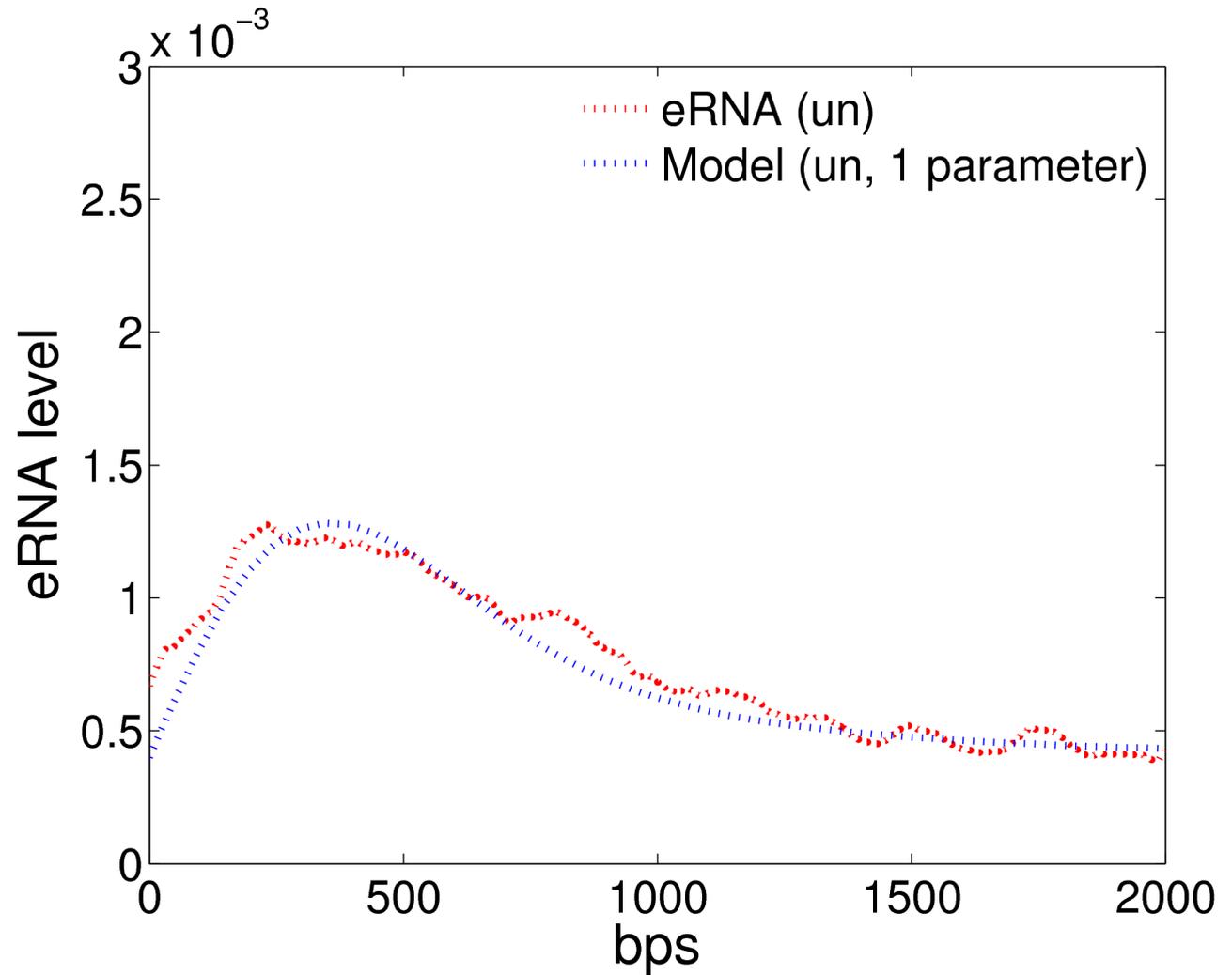
# eRNA production proportional to RNAPII



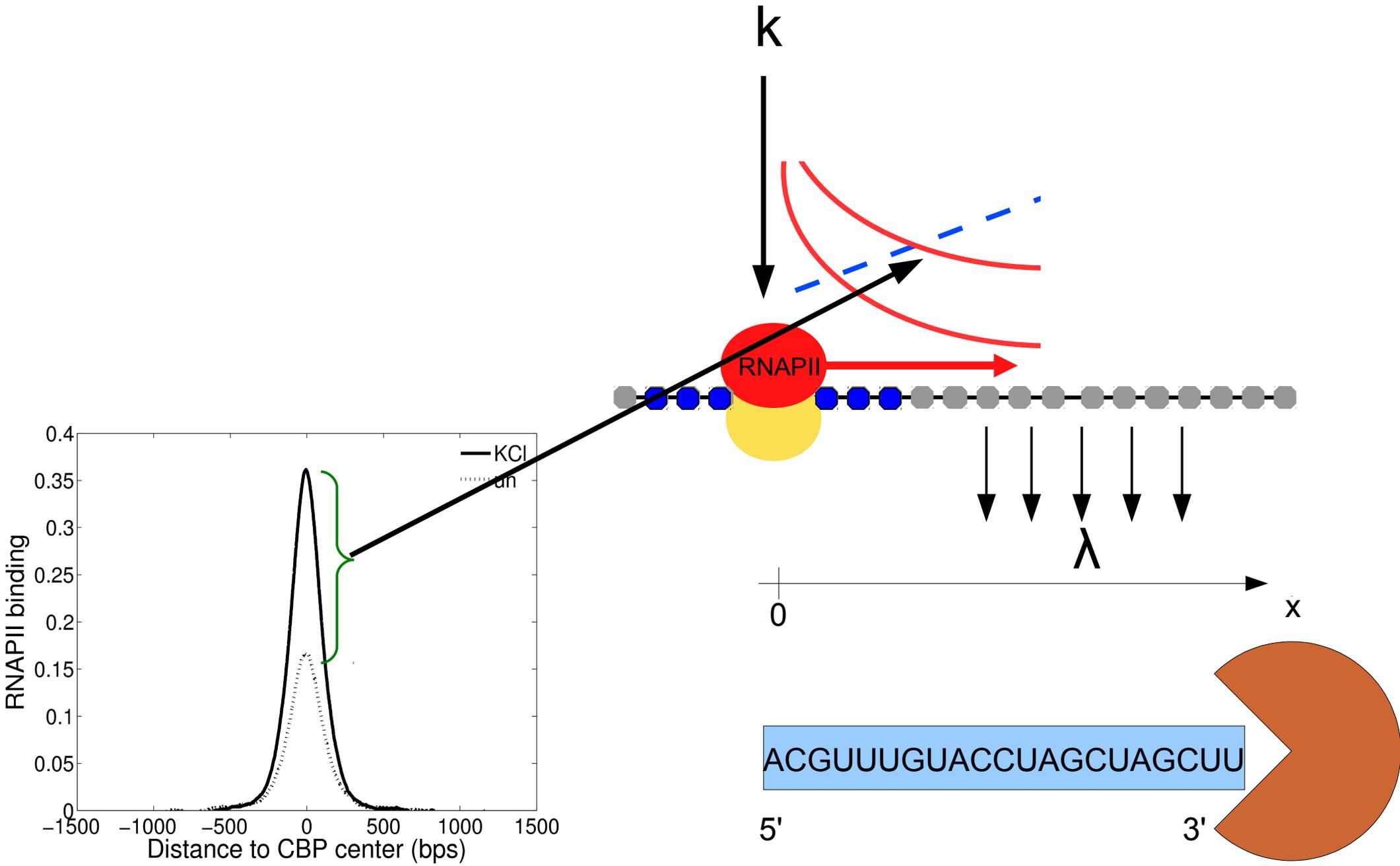
# eRNA production proportional to RNAPII



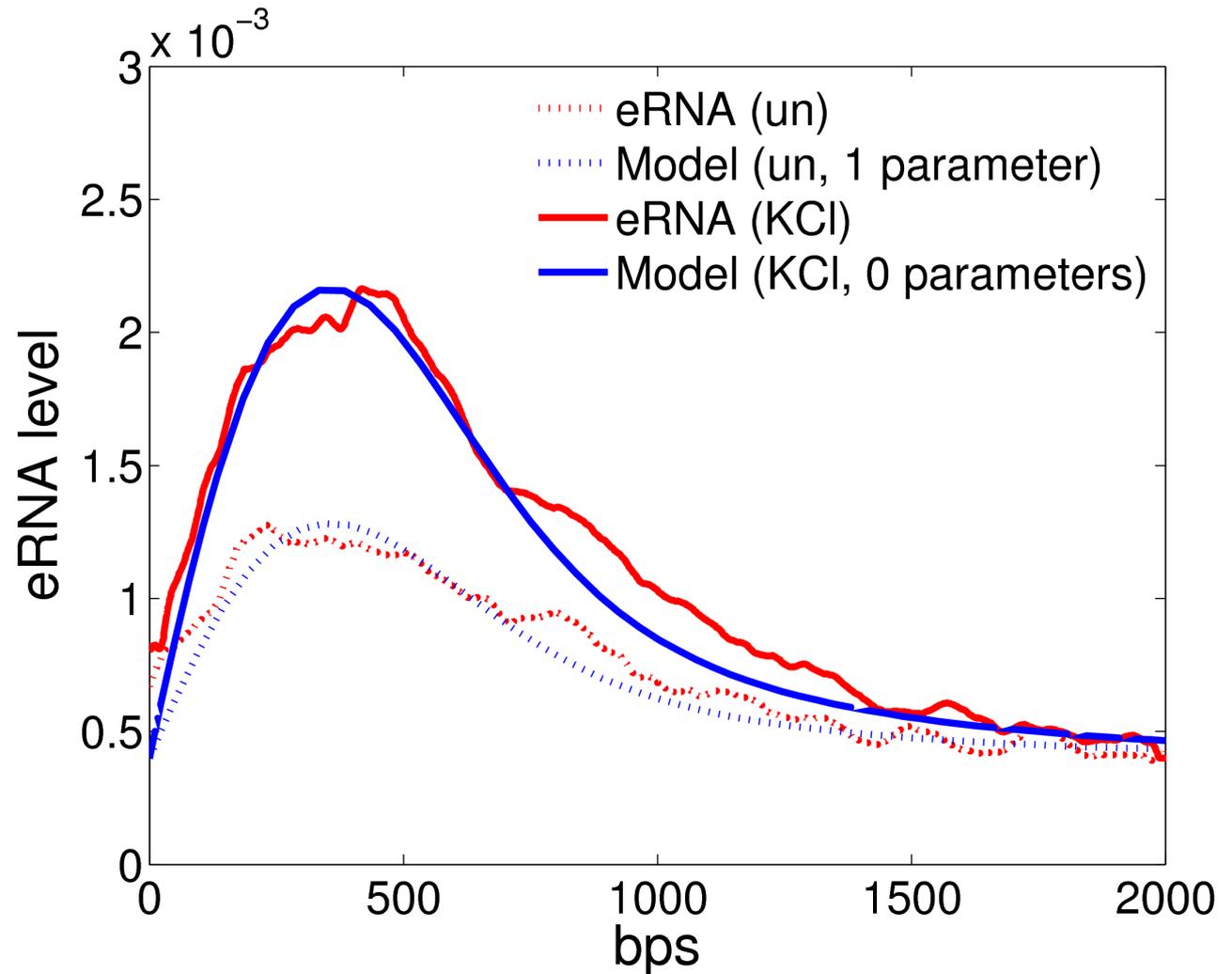
# eRNA levels can be accurately predicted



# Binding rate of RNAPII doubled after KCl



# Excellent fit for eRNA after KCl without free parameters

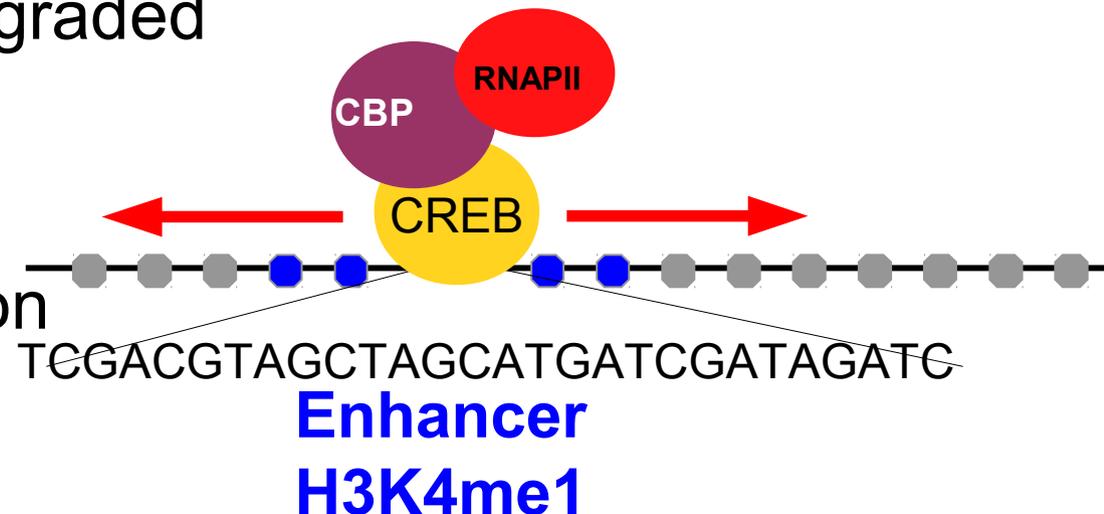


# Properties of activity-dependent enhancers

- Enriched for ~100 sequence motifs
- ChIP-seq reads predicted by sequence
- CBP binding determined by other TFs
- CBP recruits RNAPII
- RNAPII synthesizes eRNAs

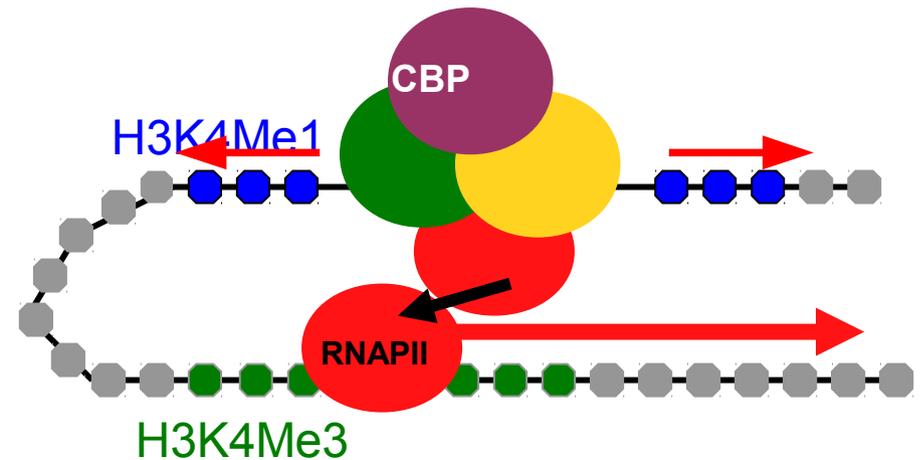
– eRNAs are rapidly degraded

– eRNA levels  
described by  
model of transcription



# What is the function of RNAPII at enhancers?

- Transcribe eRNAs
- Speed up recruitment of RNAPII at promoter

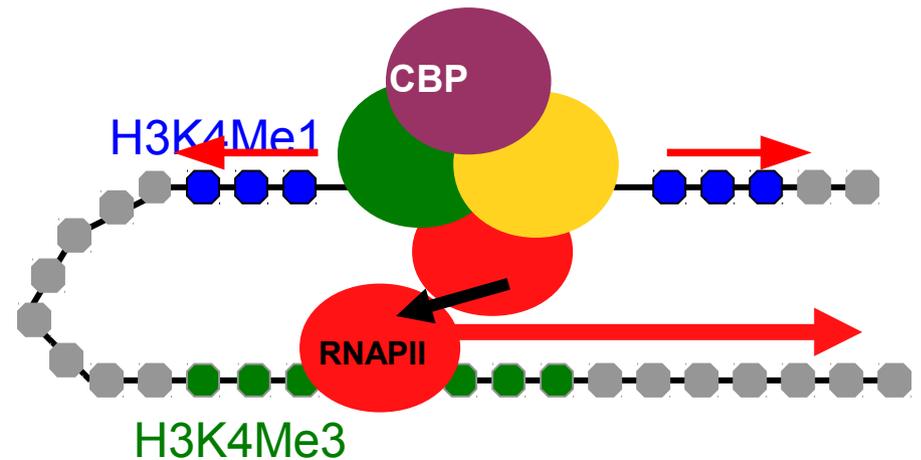


Science is always wrong. It never solves a problem without creating ten more.

-George Bernard Shaw

# Recruitment of RNAPII at the promoter

- Transcribe eRNAs
- Speed up recruitment of RNAPII at promoter



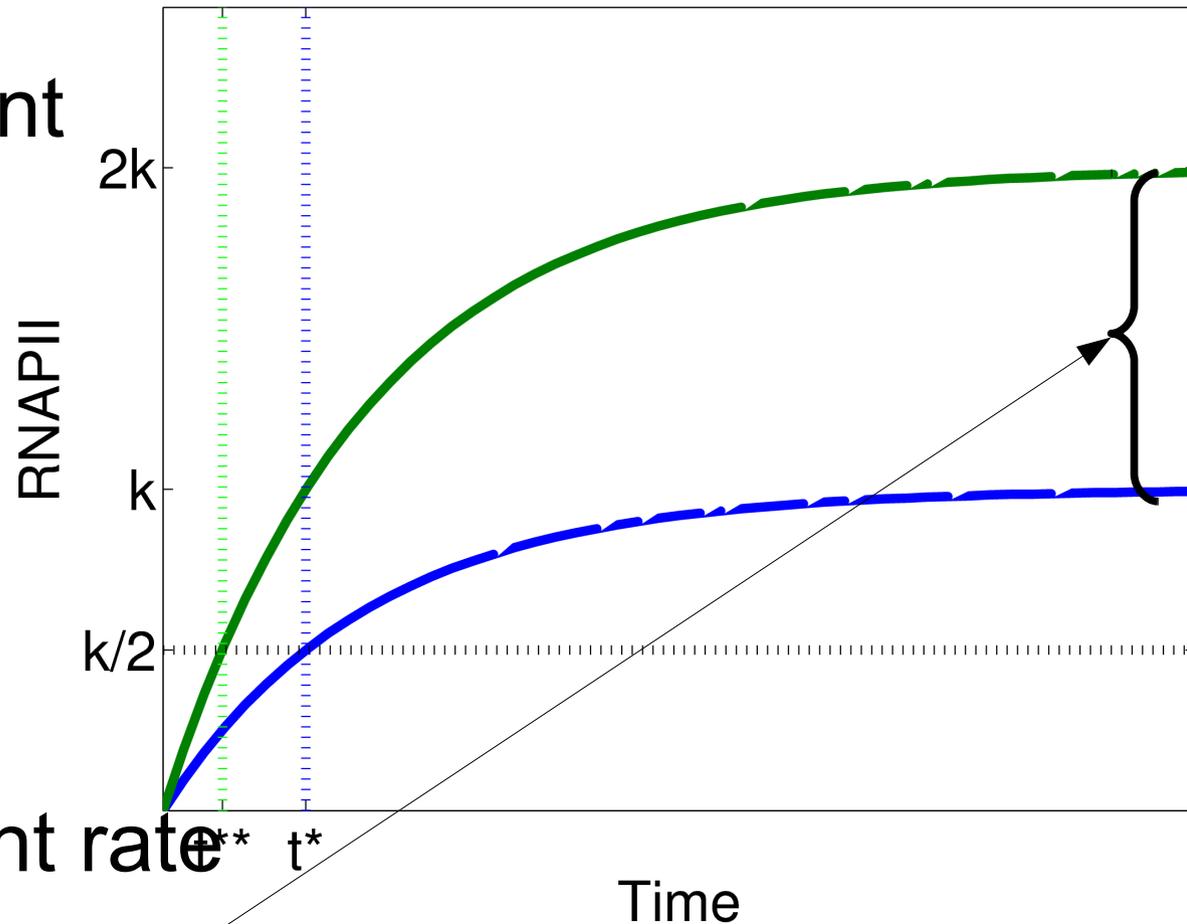
RNAPII recruitment rate

= [promoter rate]

+ N\*[contact probability]\*[enhancer rate]

# Steady state level of RNAPII is increased

- Speed up recruitment of RNAPII at promoter



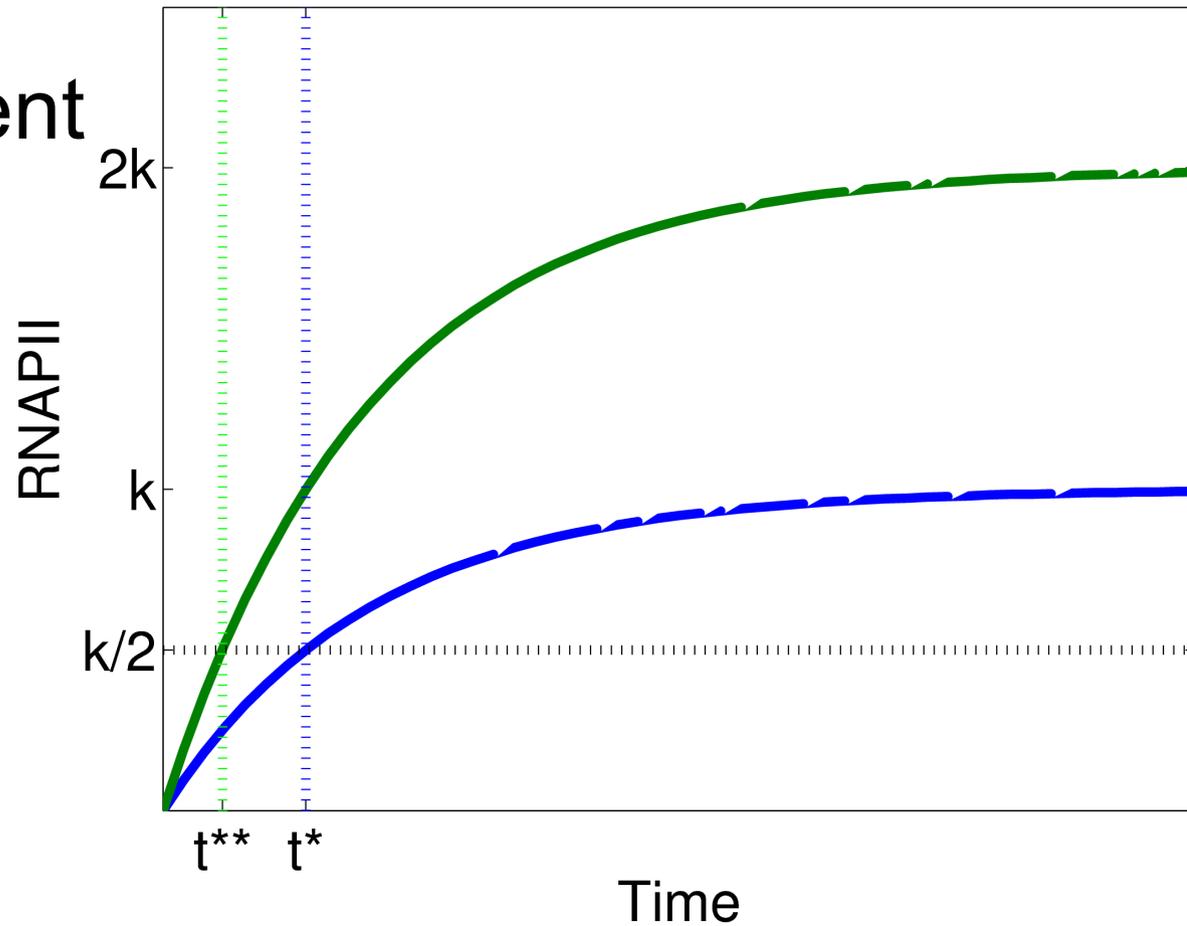
RNAPII recruitment rate  $e^{**}$

= [promoter rate]

+  $N^*$ [contact probability]\*[enhancer rate]

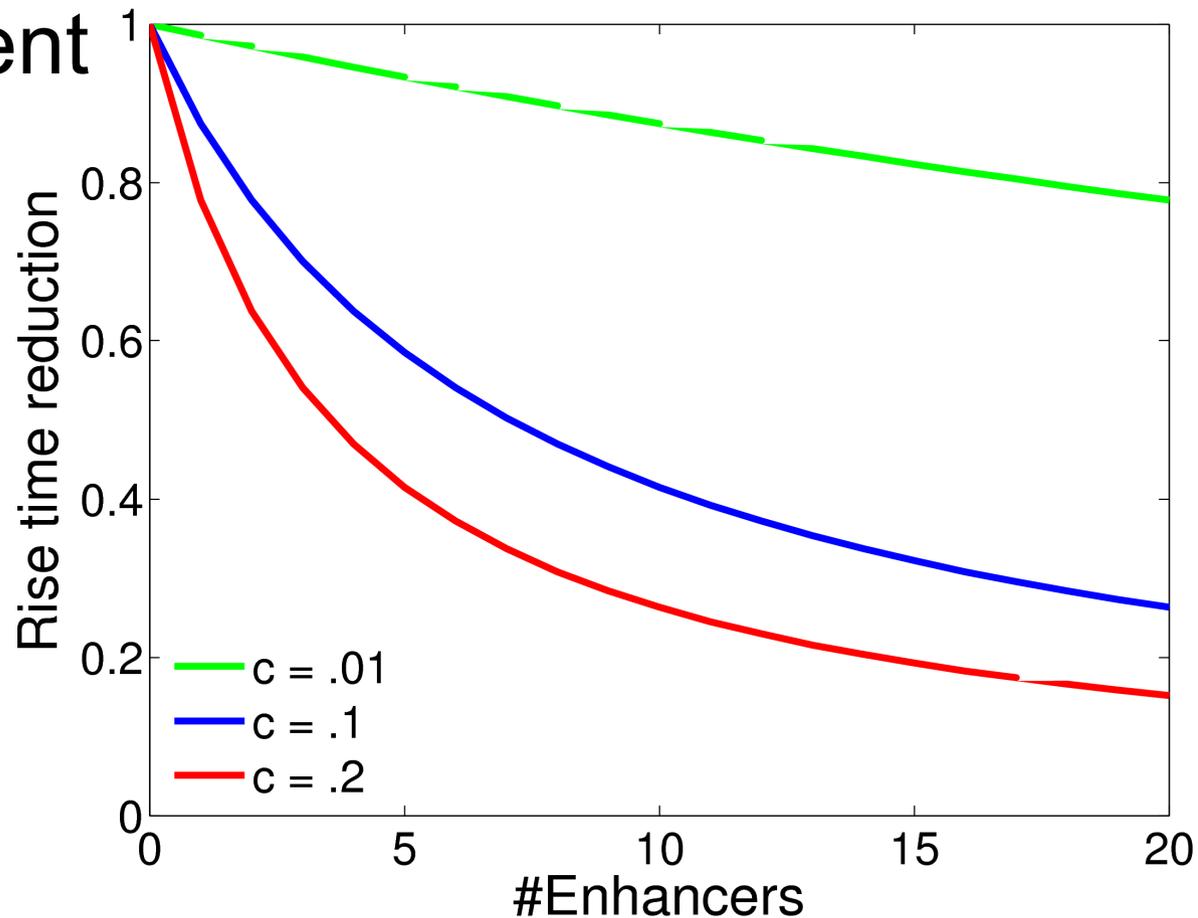
# Rise time is reduced

- Speed up recruitment of RNAPII at promoter



# Significant speed-up with ~5 enhancers

- Speed up recruitment of RNAPII at promoter



# Enhancers may reduce the noise in RNAPII

- Speed up recruitment of RNAPII at promoter
- Reduce noise

RNAPII recruitment rate

= [promoter rate]

+  $N \cdot [\text{contact probability}] \cdot [\text{enhancer rate}]$

# Reduction of noise proportional to the number of enhancers

- Speed up recruitment of RNAPII at promoter
- Reduce noise

RNAPII recruitment rate

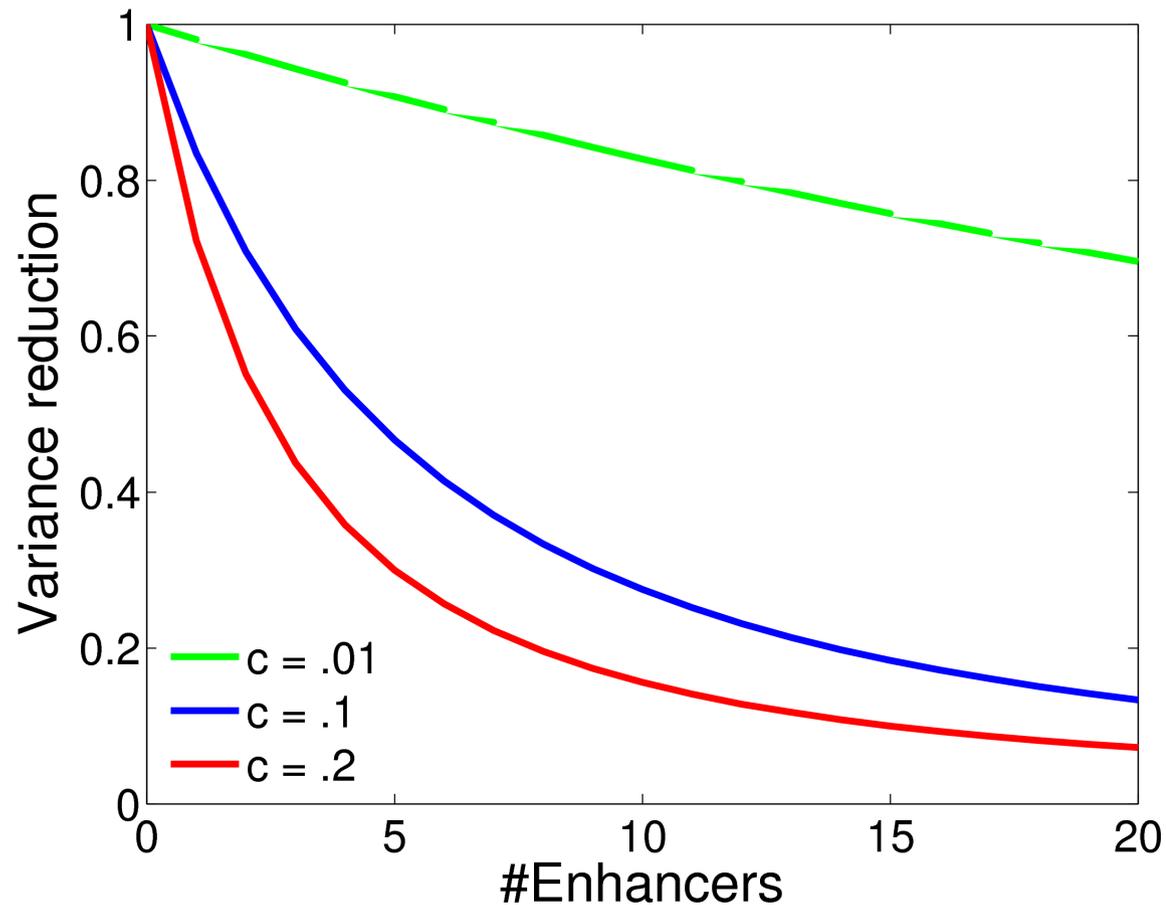
= [promoter rate]

+ N\*[contact probability]\*[enhancer rate]

$$\frac{\text{Variance strong promoter}}{\text{Variance weak promoter with enhancers}} = \frac{\text{Var}[(1 + Nc)k]}{\text{Var}[k] + N\text{Var}[ck]} = \frac{(1 + Nc)^2\text{Var}[k]}{(1 + Nc^2)\text{Var}[k]} \sim N$$

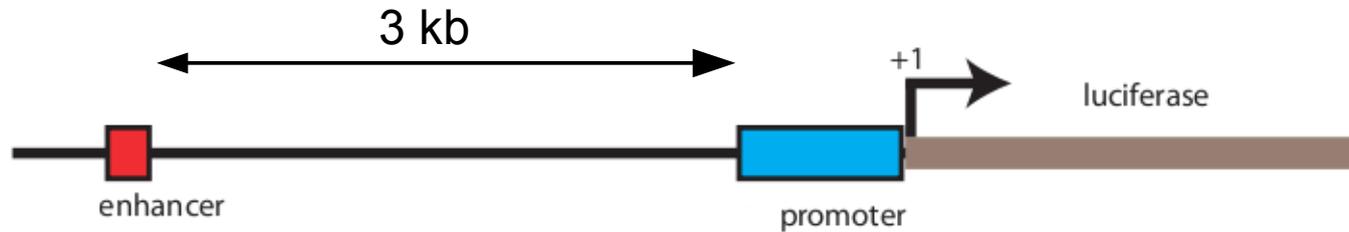
~50% reduction of noise with ~5 enhancers

- Speed up recruitment of RNAPII at promoter
- Reduce noise



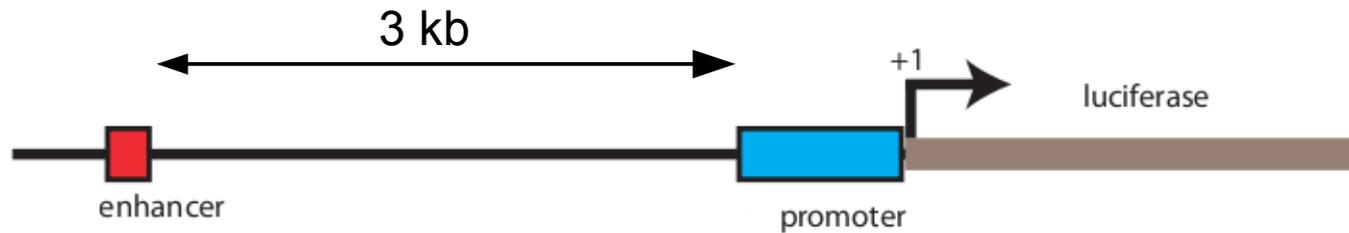
$$\frac{\text{Variance strong promoter}}{\text{Variance weak promoter with enhancers}} = \frac{\text{Var}[(1 + Nc)k]}{\text{Var}[k] + N\text{Var}[ck]} = \frac{(1 + Nc)^2 \text{Var}[k]}{(1 + Nc^2) \text{Var}[k]} \sim N$$

# Do eRNAs enhance gene expression?



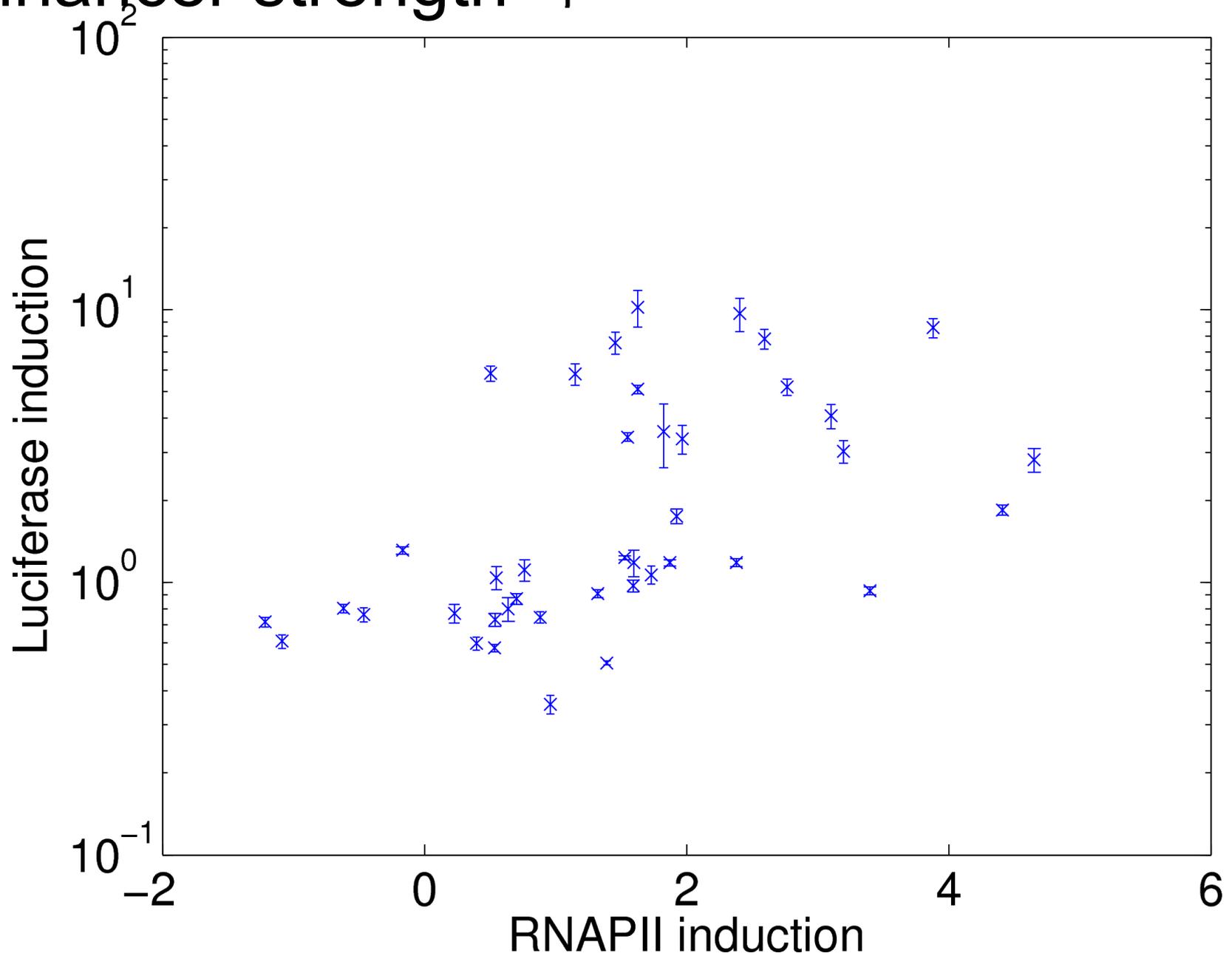
- Luciferase construct to test enhancer function

# Do eRNAs enhance gene expression?

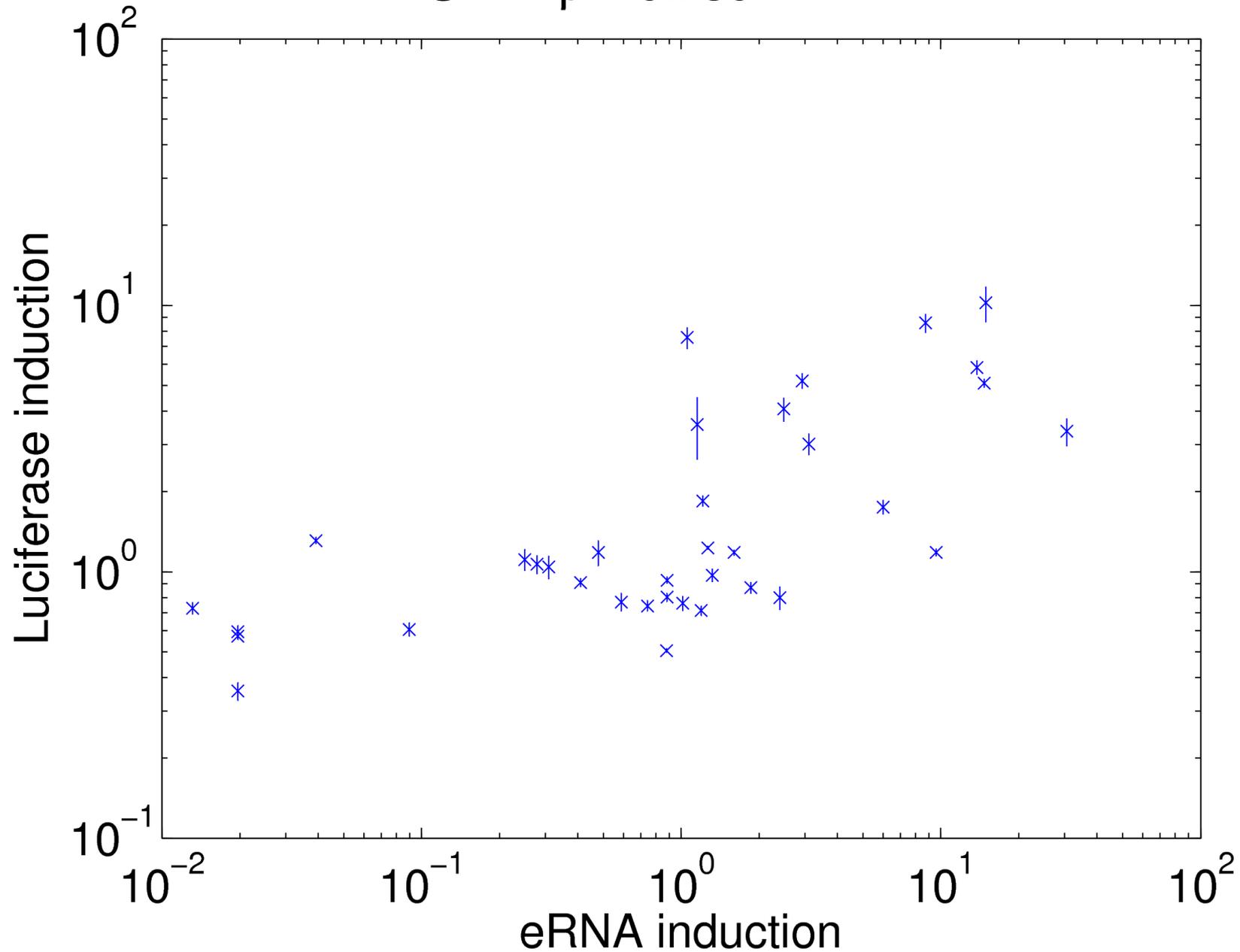


- Luciferase construct to test enhancer function
  - Narp gene

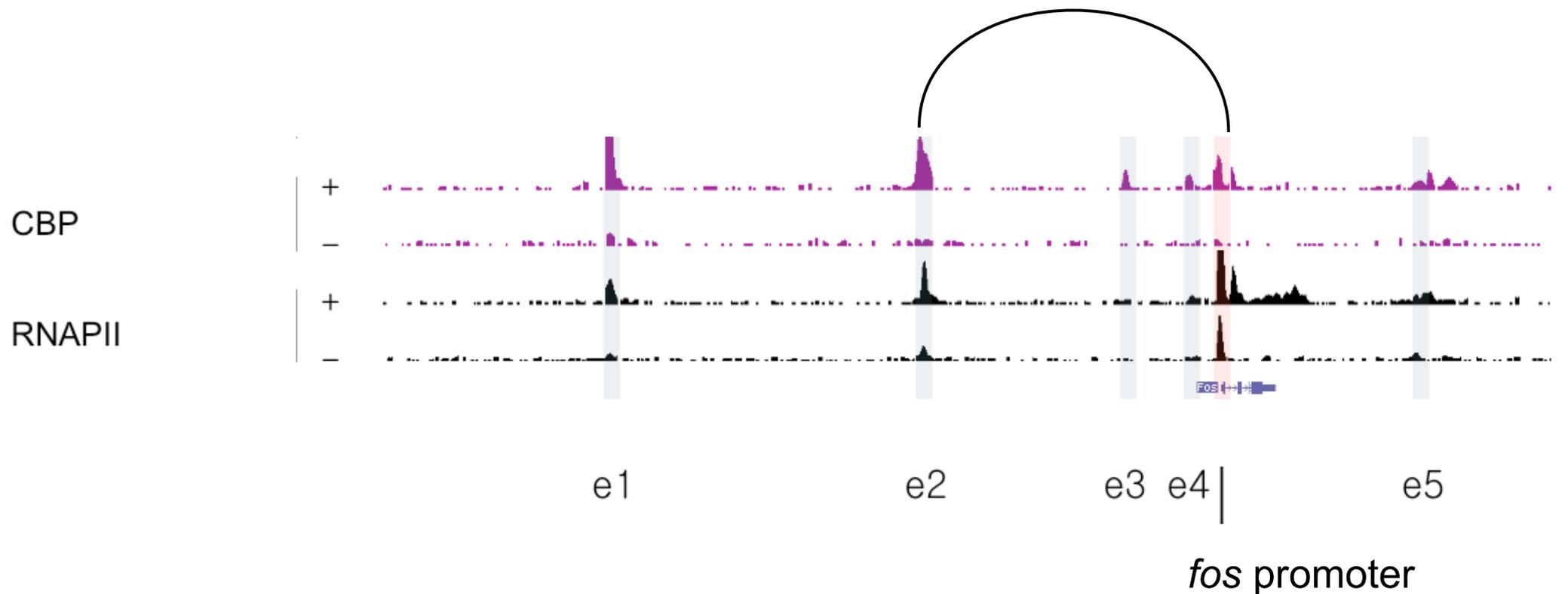
RNAPII induction is weakly correlated with enhancer strength  $\rho = 0.50$



eRNA induction is strongly correlated with enhancer strength  $\rho = 0.739$

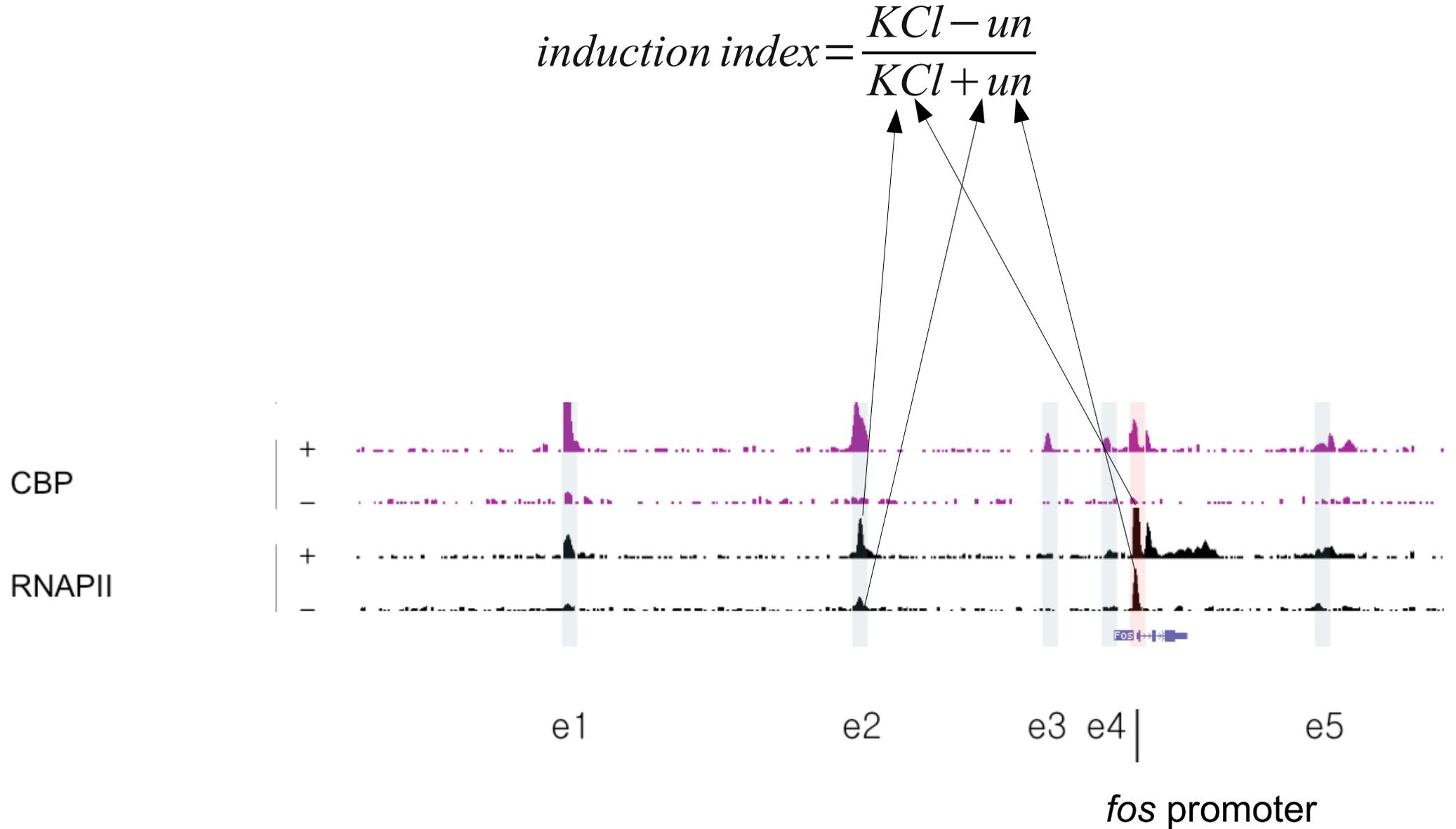


# Is eRNA induction correlated with mRNA induction in vivo?



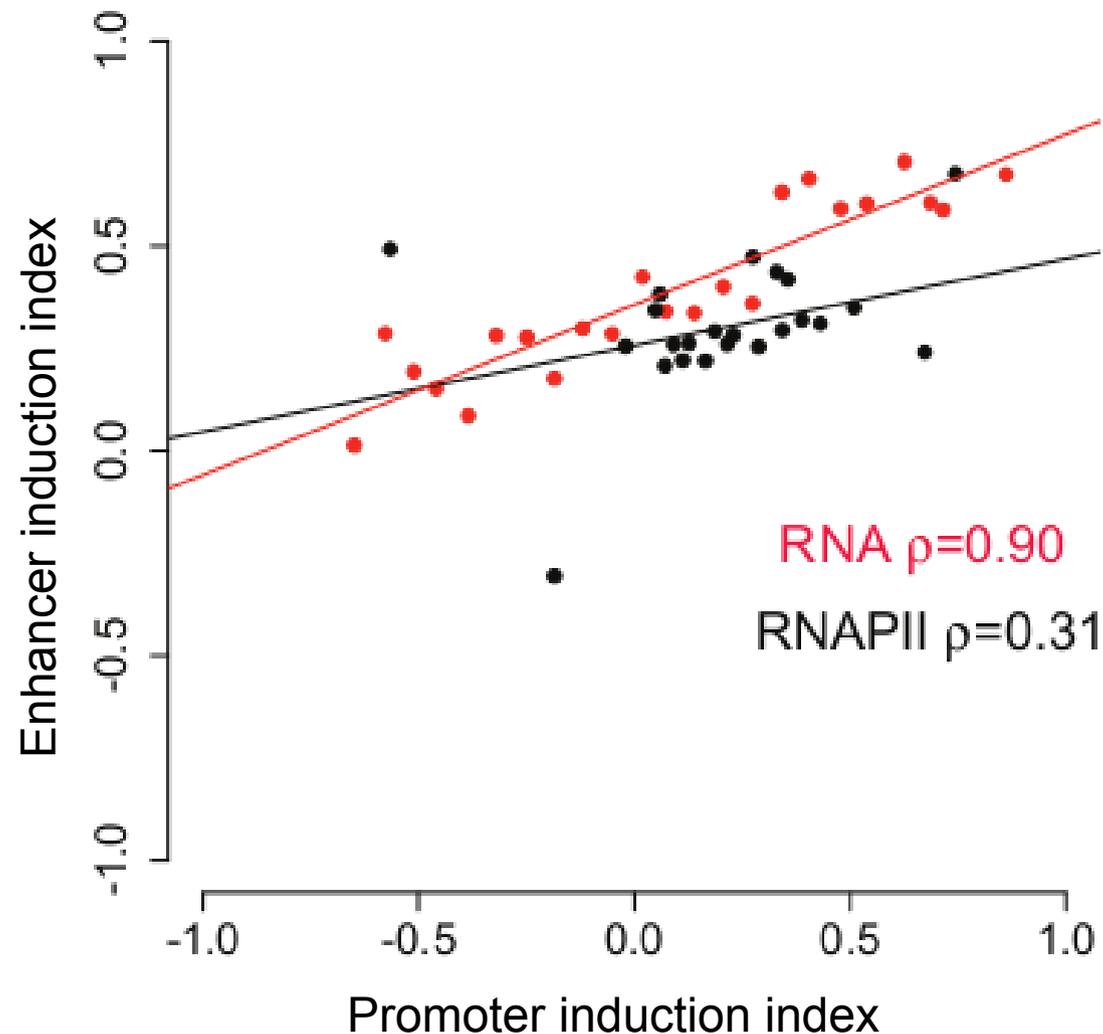
# Normalized induction index to compare induction of RNAPII and transcription

$$\text{induction index} = \frac{KCl - un}{KCl + un}$$



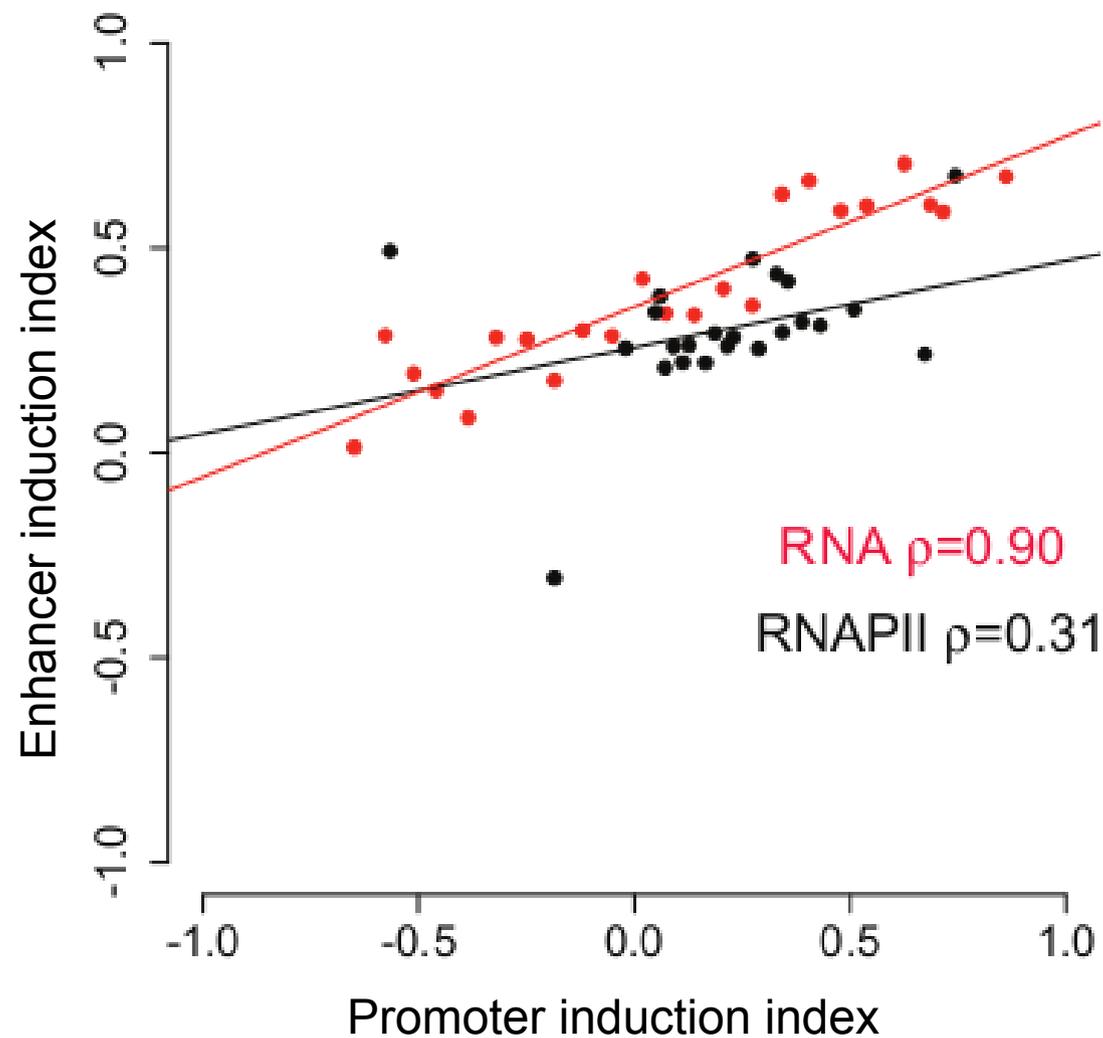
# eRNA induction is correlated with induction of nearby mRNAs

$$\text{induction index} = \frac{KCl - un}{KCl + un}$$

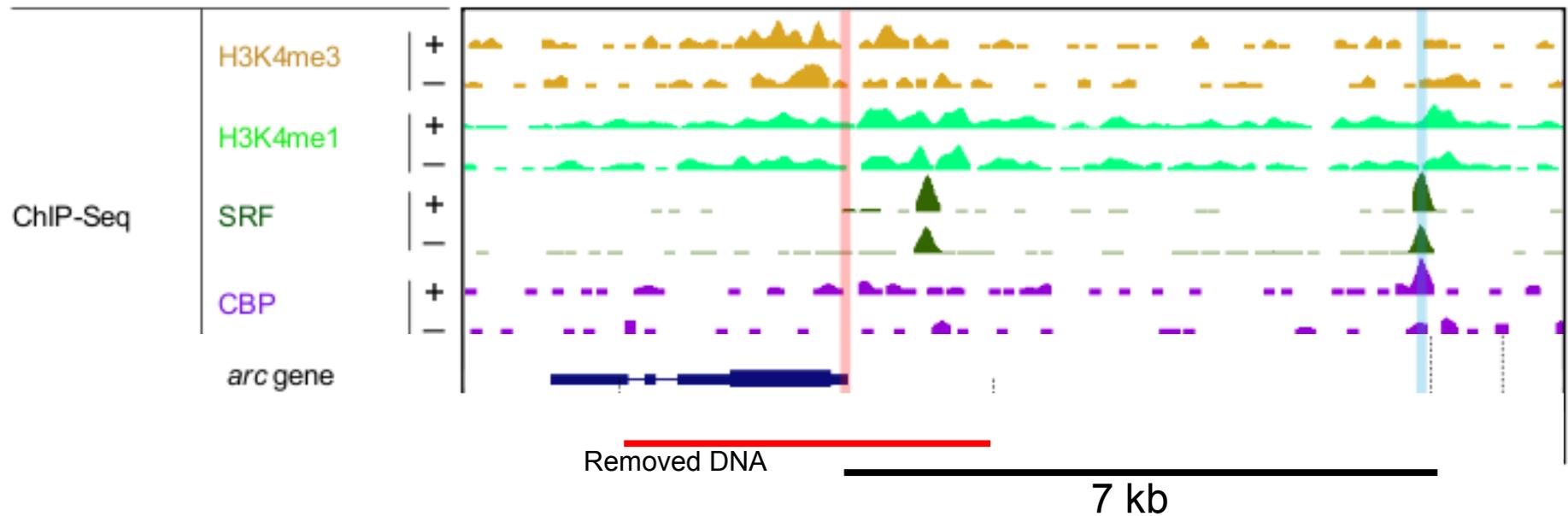


# Do eRNAs depend on mRNAs?

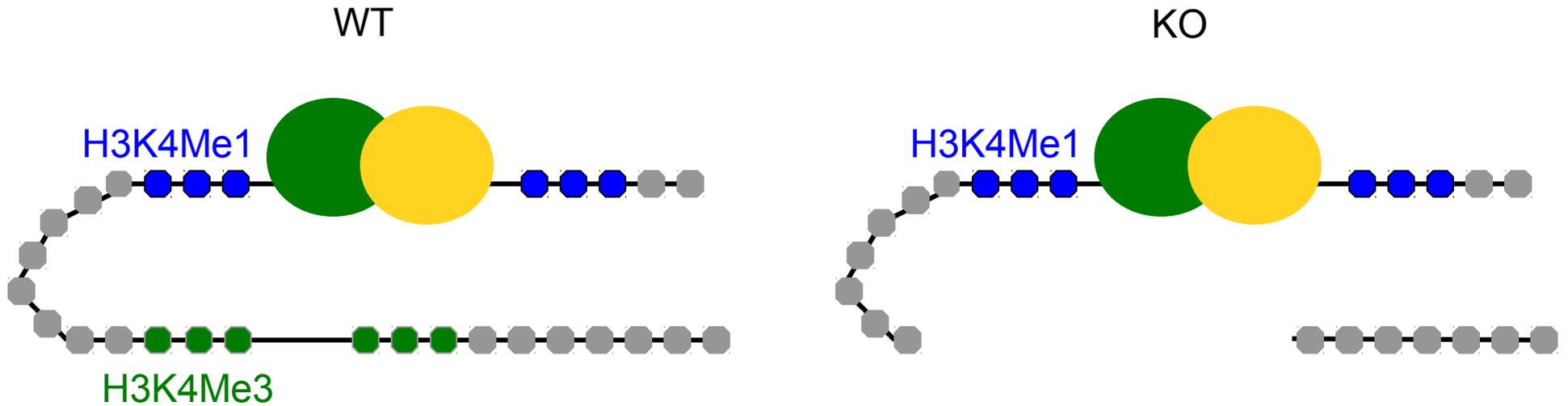
$$\text{induction index} = \frac{KCl - un}{KCl + un}$$



# Knock-out experiment of the *arc*-promoter

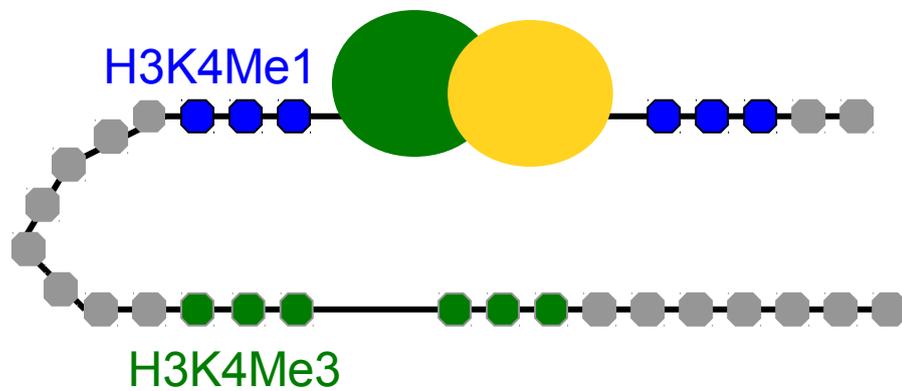


# Are eRNAs independent of the promoter?

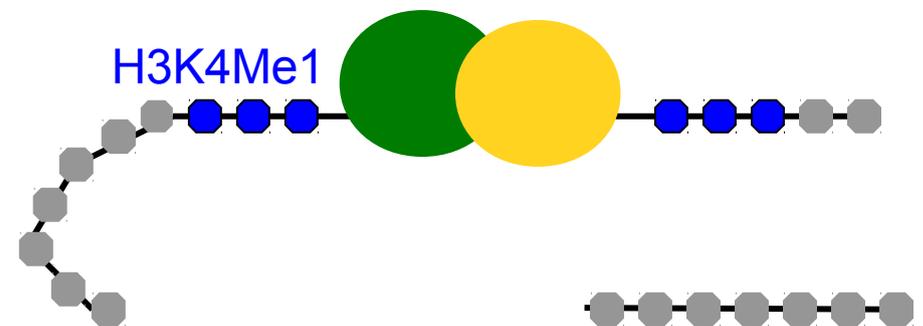


# RNAPII increases 5-fold in both WT and KO

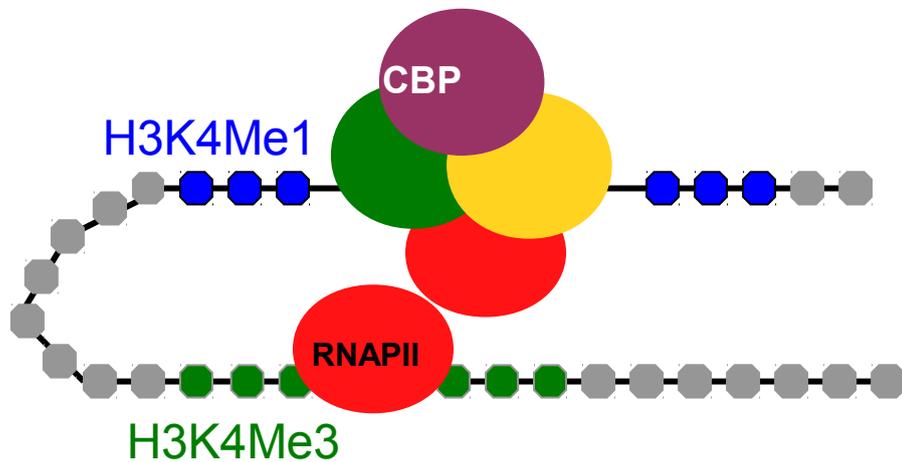
WT



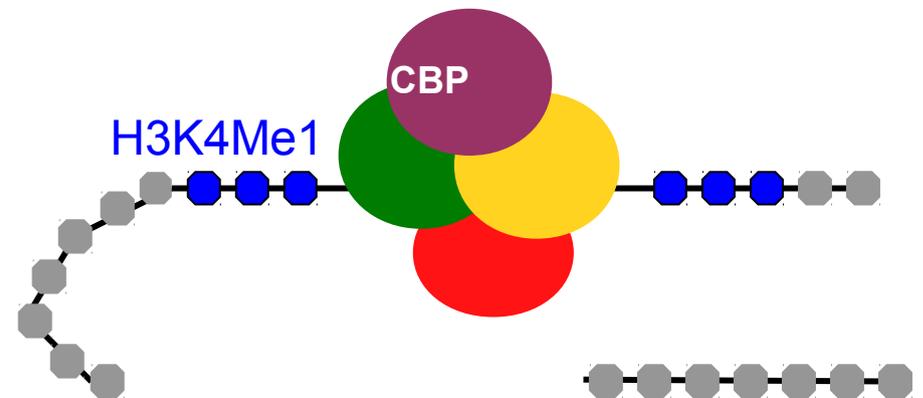
KO



CBP

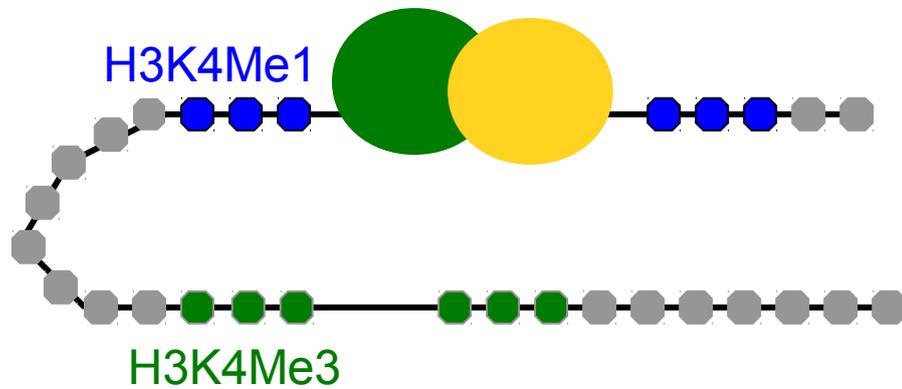


CBP

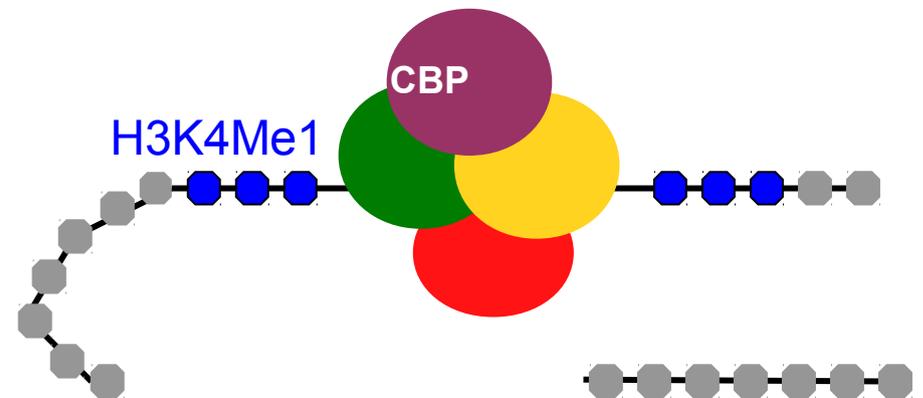
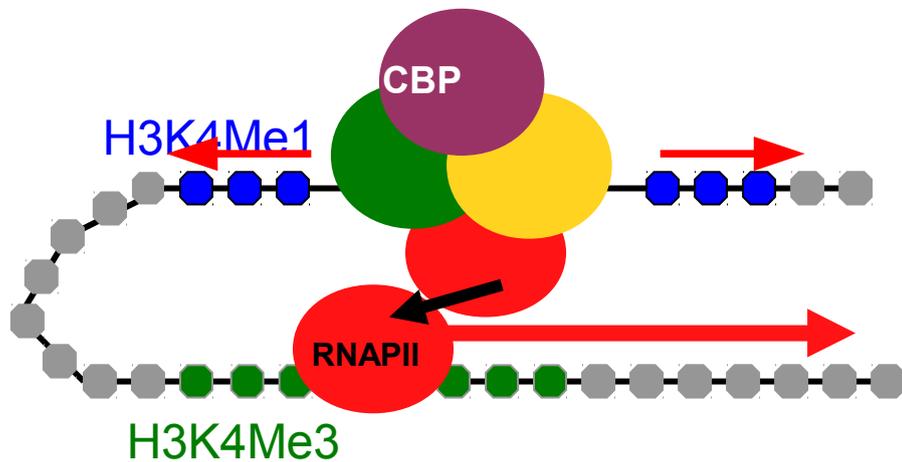
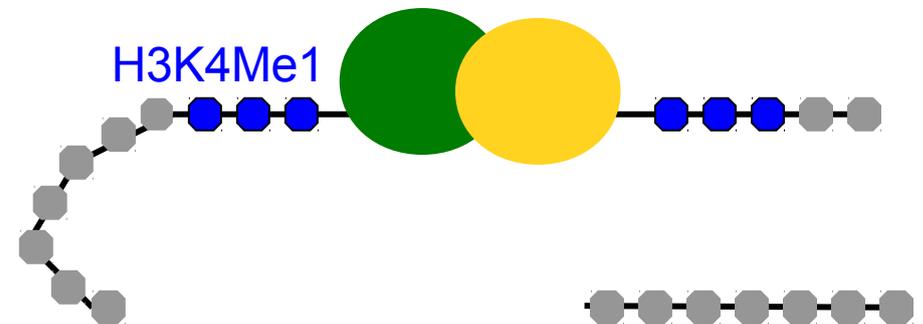


# eRNAs are not present in KO

WT



KO



# Summary

- Identified ~12k activity-dependent enhancers
- Discovered and quantified novel mechanisms
  - Identified enriched motifs and bound TFs

# Summary

- Identified ~12k activity-dependent enhancers
- Discovered and quantified novel mechanisms

- Identified enriched motifs and bound TFs
- Combinatorial code for CBP affinity
- Recruitment of RNAPII at enhancers
  - Faster recruitment to promoter
  - Reduce noise
- Transcription at enhancers
  - Properties of eRNA
  - Model of RNAPII and eRNA levels
  - Interaction with promoter necessary

---

# Widespread transcription at neuronal activity-regulated enhancers

Tae-Kyung Kim<sup>1\*†</sup>, Martin Hemberg<sup>2\*</sup>, Jesse M. Gray<sup>1\*</sup>, Allen M. Costa<sup>1</sup>, Daniel M. Bear<sup>1</sup>, Jing Wu<sup>3</sup>, David A. Harmin<sup>1,4</sup>, Mike Laptewicz<sup>1</sup>, Kellie Barbara-Haley<sup>5</sup>, Scott Kuersten<sup>6</sup>, Eirene Markenscoff-Papadimitriou<sup>1†</sup>, Dietmar Kuhl<sup>7</sup>, Haruhiko Bito<sup>8</sup>, Paul F. Worley<sup>3</sup>, Gabriel Kreiman<sup>2</sup> & Michael E. Greenberg<sup>1</sup>

# eRNAs have been found in other cell types

doi:10.1038/nature09033

nature

ARTICLES

## Widespread transcription at neuronal activity-regulated enhancers

Tae-Kyung Kim<sup>1\*†</sup>, Martin Hemberg<sup>2\*</sup>, Jesse M. Gray<sup>1\*</sup>, Allen M. Costa<sup>1</sup>, Daniel M. Bear<sup>1</sup>, Jing Wu<sup>3</sup>, David A. Harmin<sup>1,4</sup>, Mike Laptewicz<sup>1</sup>, Kellie Barbara-Haley<sup>5</sup>, Scott Kuersten<sup>6</sup>, Eirene Markenscoff-Papadimitriou<sup>1†</sup>, Dietmar Kuhl<sup>7</sup>, Haruhiko Bito<sup>8</sup>, Paul F. Worley<sup>3</sup>, Gabriel Kreiman<sup>2</sup> & Michael E. Greenberg<sup>1</sup>

## Histone H3K27ac separates active from poised enhancers and predicts developmental state

Menno P. Creyghton<sup>a,1</sup>, Albert W. Cheng<sup>a,b,1</sup>, G. Grant Welstead<sup>a</sup>, Tristan Kooistra<sup>c,d</sup>, Bryce W. Carey<sup>a,e</sup>, Eveline J. Steine<sup>a,e</sup>, Jacob Hanna<sup>a</sup>, Michael A. Lodato<sup>a,e</sup>, Garrett M. Frampton<sup>a,e</sup>, Phillip A. Sharp<sup>d,e</sup>, Laurie A. Boyer<sup>e</sup>, Richard A. Young<sup>a,e</sup>, and Rudolf Jaenisch<sup>a,e,2</sup>

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PLoS BIOLOGY

## A Large Fraction of Extragenic RNA Pol II Transcription Sites Overlap Enhancers

Francesca De Santa<sup>1,3</sup>, Iros Barozzi<sup>1,3</sup>, Flore Mietton<sup>1,3</sup>, Serena Ghisletti<sup>1</sup>, Sara Polletti<sup>1</sup>, Betsabeh Khoramian Tusi<sup>1</sup>, Heiko Muller<sup>1</sup>, Jiannis Ragoussis<sup>2</sup>, Chia-Lin Wei<sup>3</sup>, Gioacchino Natoli<sup>1\*</sup>

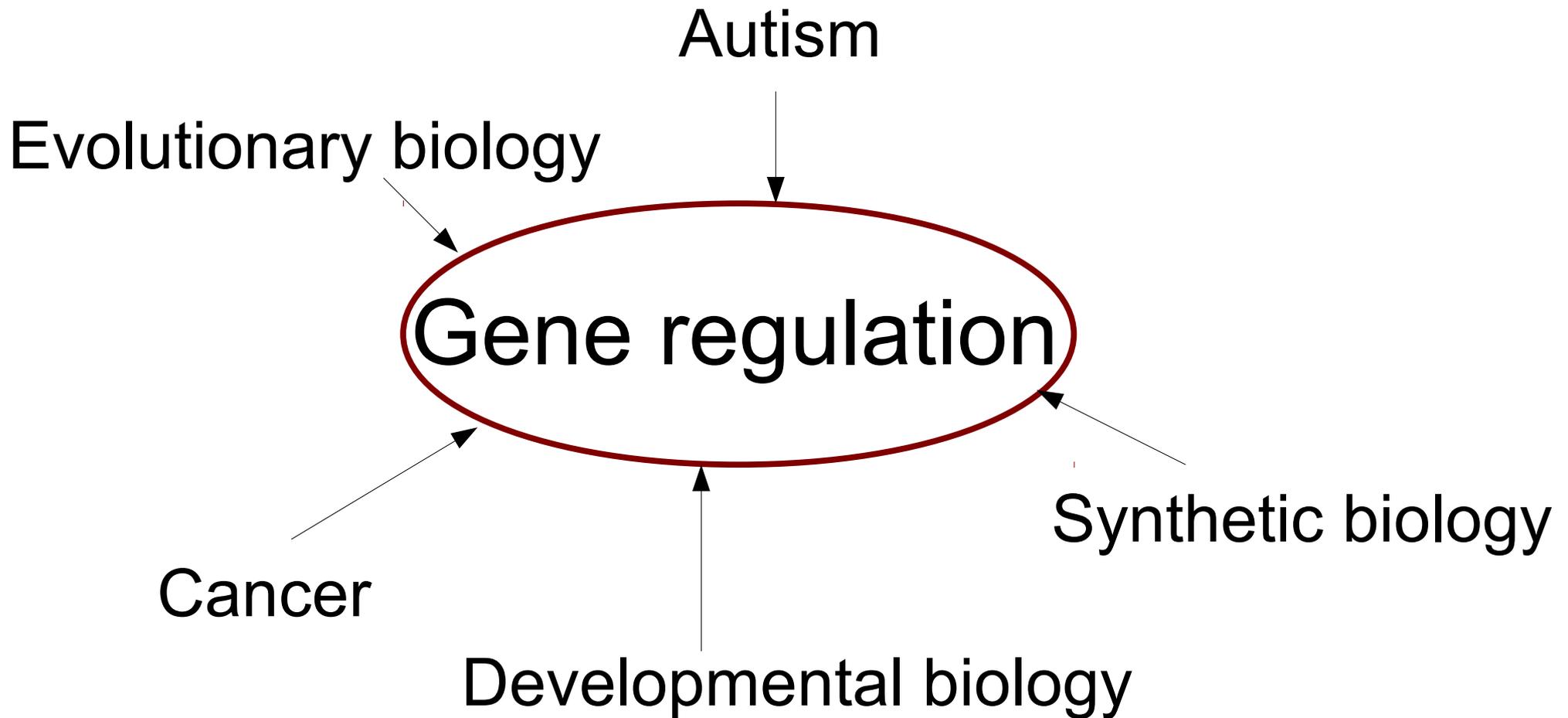
LETTER

doi:10.1038/nature09692

## A unique chromatin signature uncovers early developmental enhancers in humans

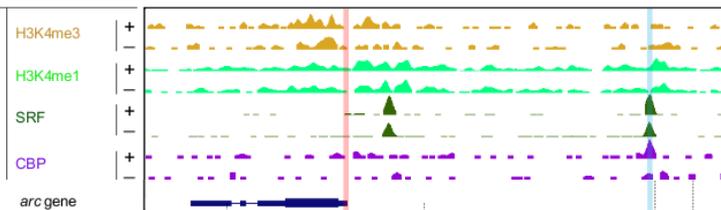
Alvaro Rada-Iglesias<sup>1</sup>, Ruchi Bajpai<sup>1</sup>, Tomek Swigut<sup>1</sup>, Samantha A. Brugmann<sup>1</sup>, Ryan A. Flynn<sup>1</sup> & Joanna Wysocka<sup>1,2</sup>

# Why is gene regulation important?

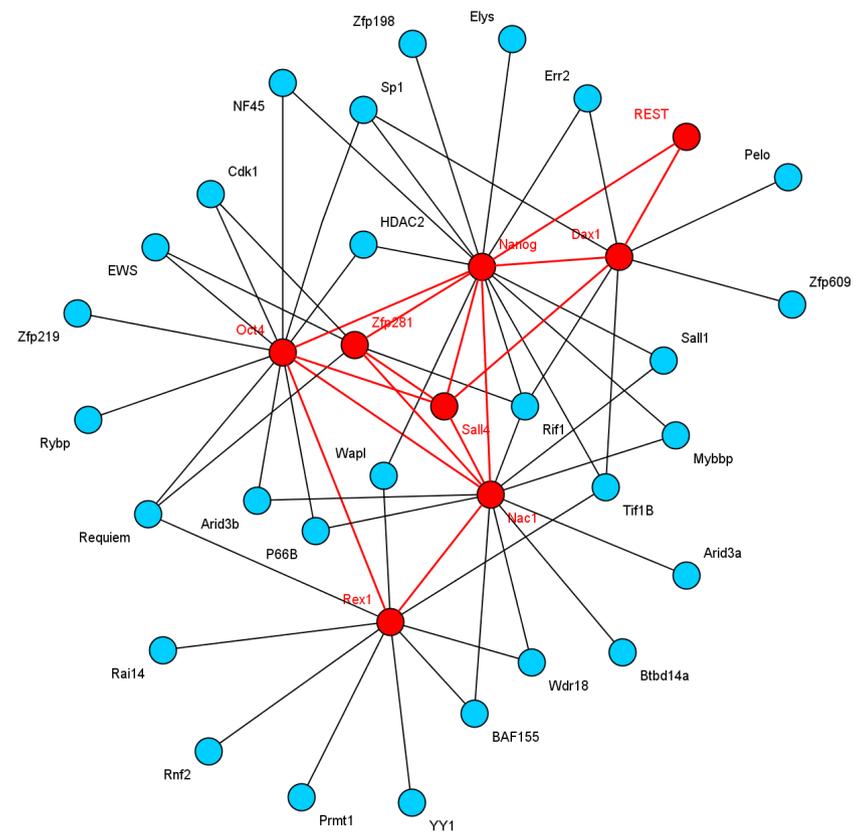


# Future Work: Organizing principles of the genome

- Use genome-wide data to develop systems biology type models of gene regulation

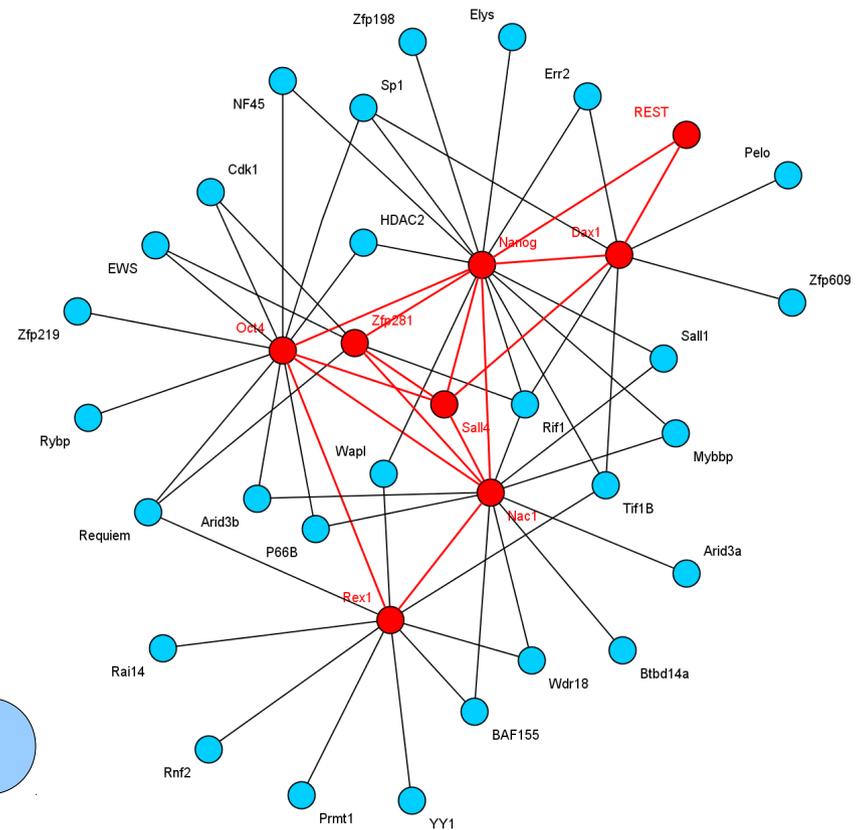
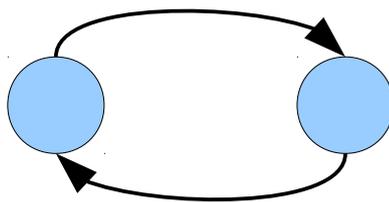
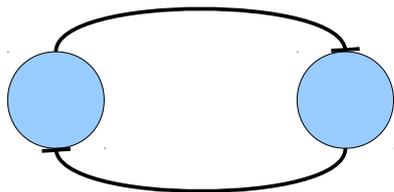


+

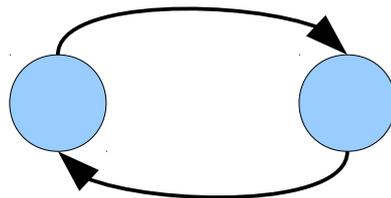
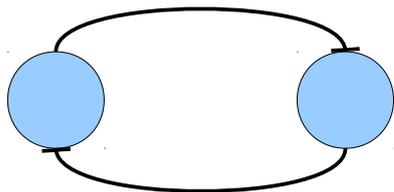
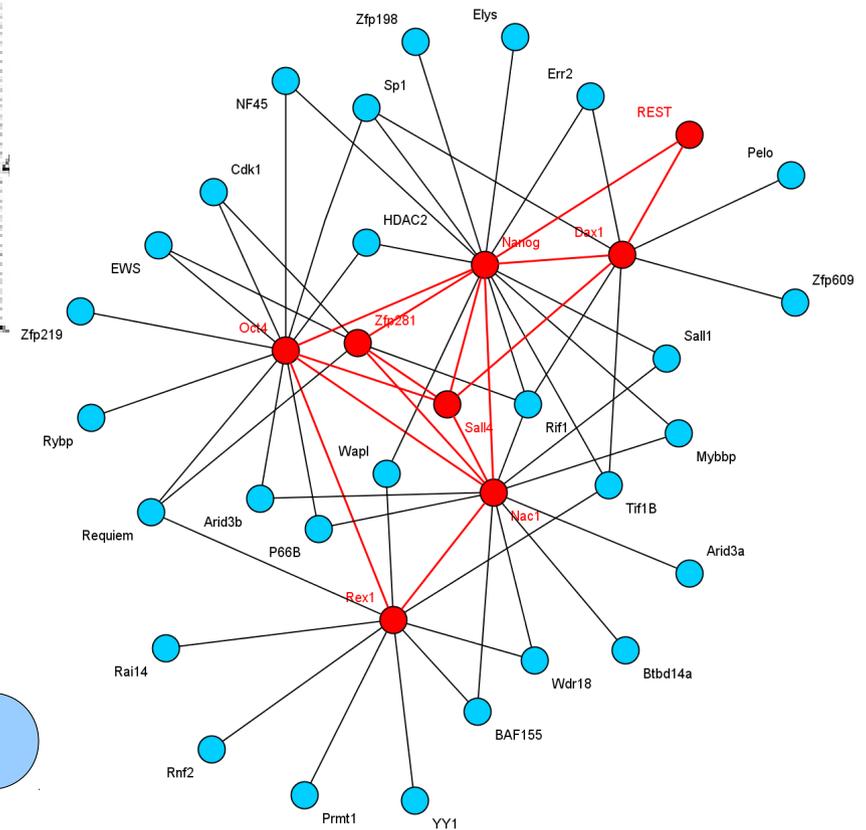
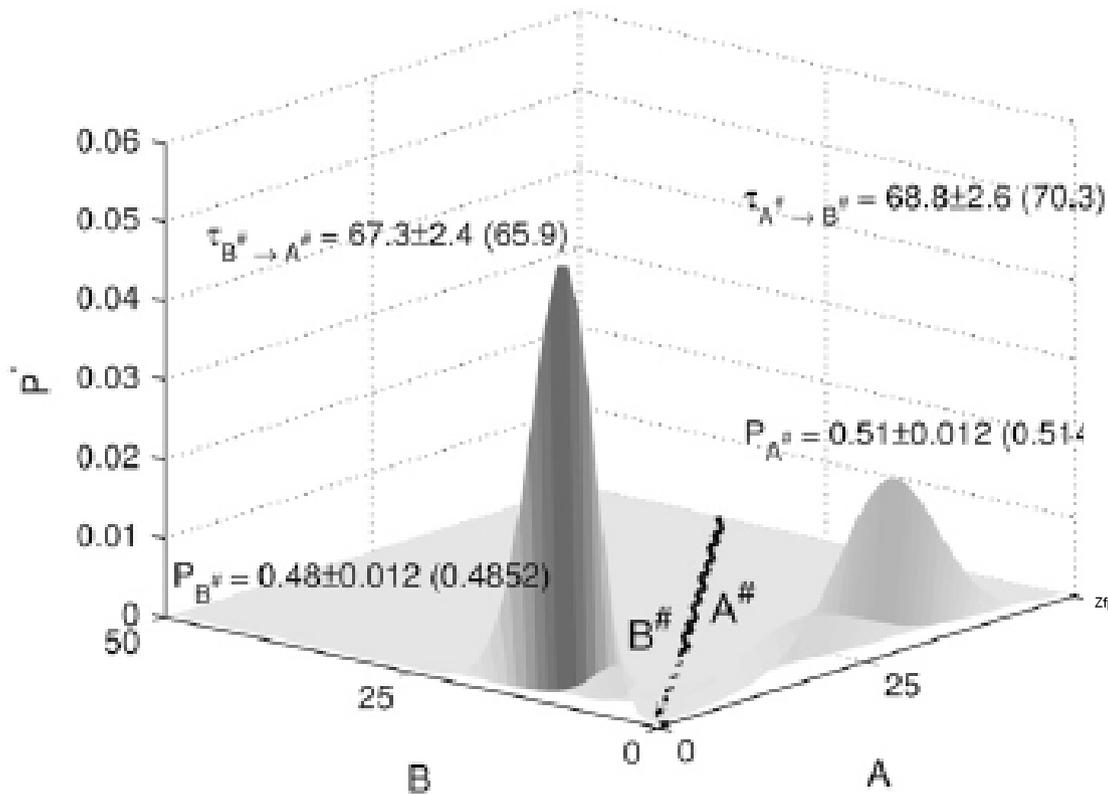




# What are the dynamical properties of gene regulatory networks?



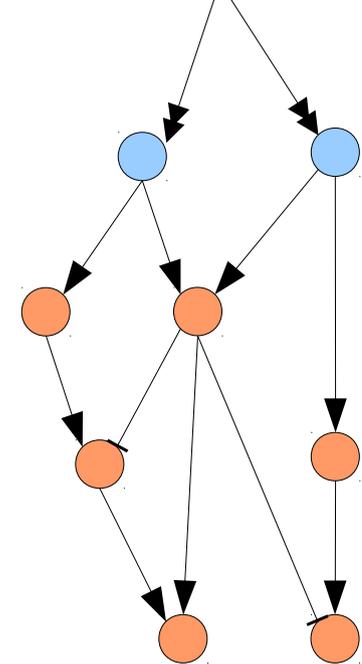
# What is the role of noise in gene regulation?



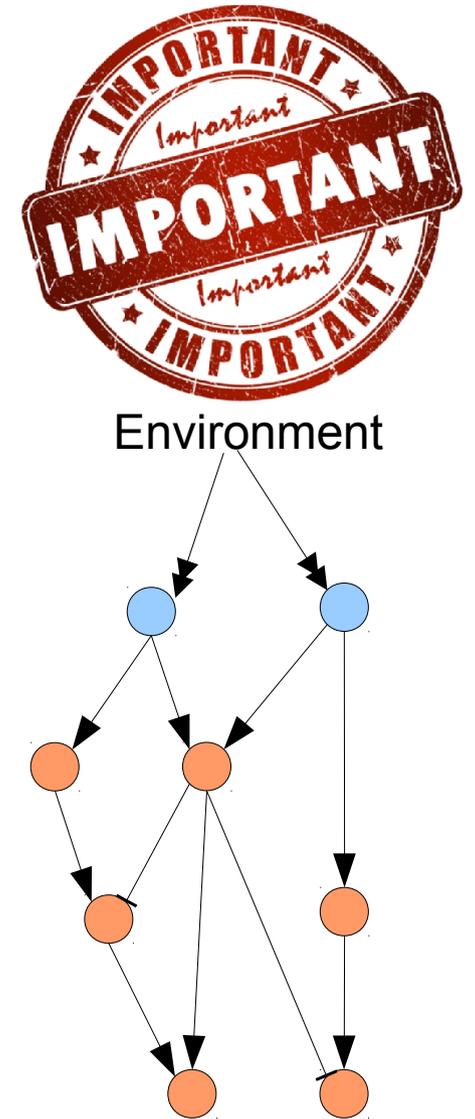
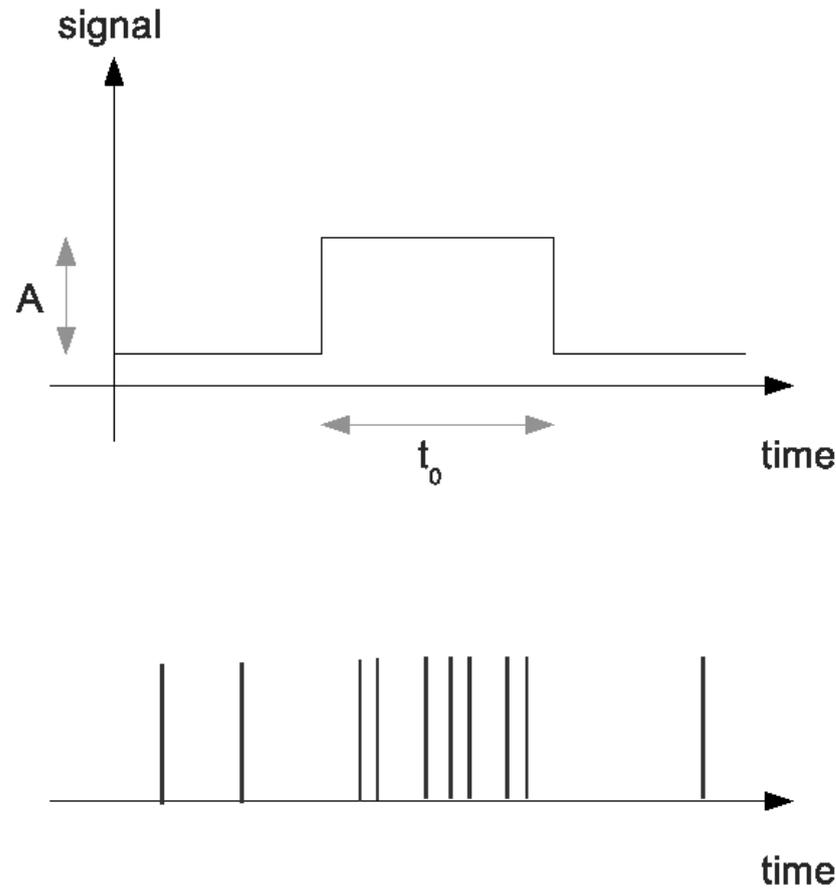
# How is information propagated in gene regulatory networks?



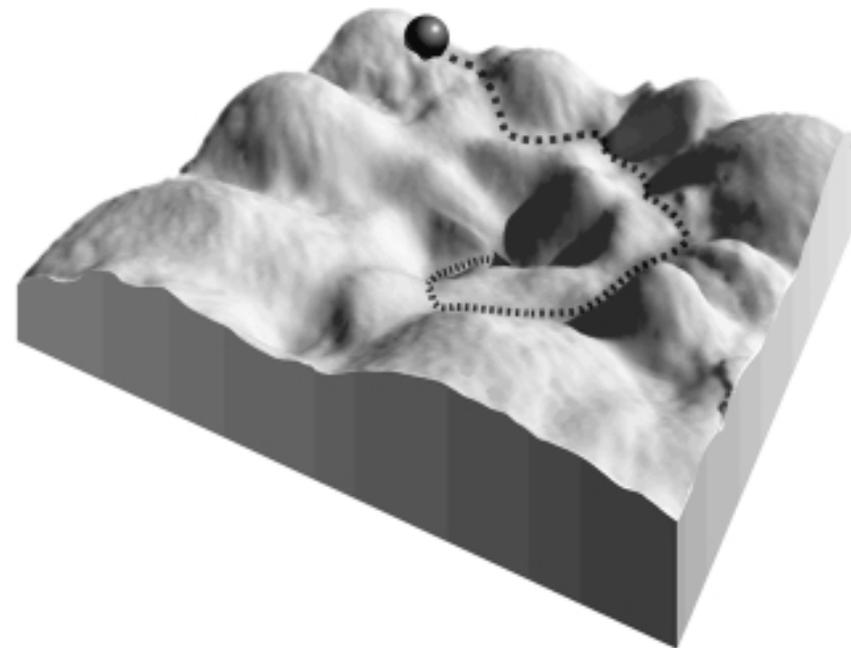
Environment



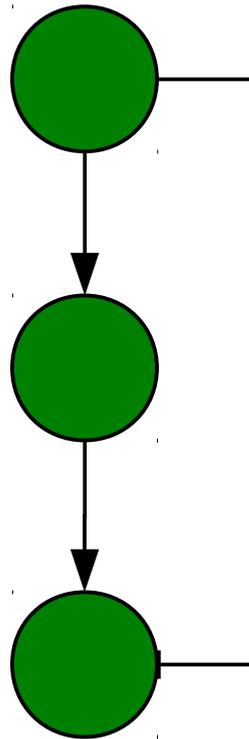
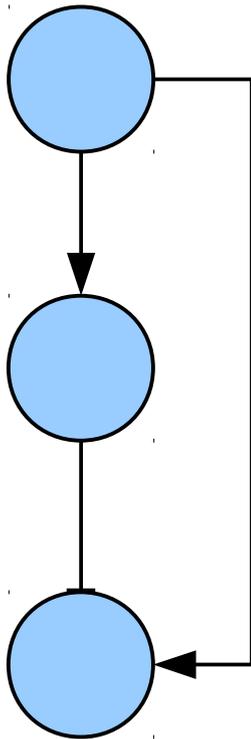
# How can the input signals be inferred from observing the mRNA levels?



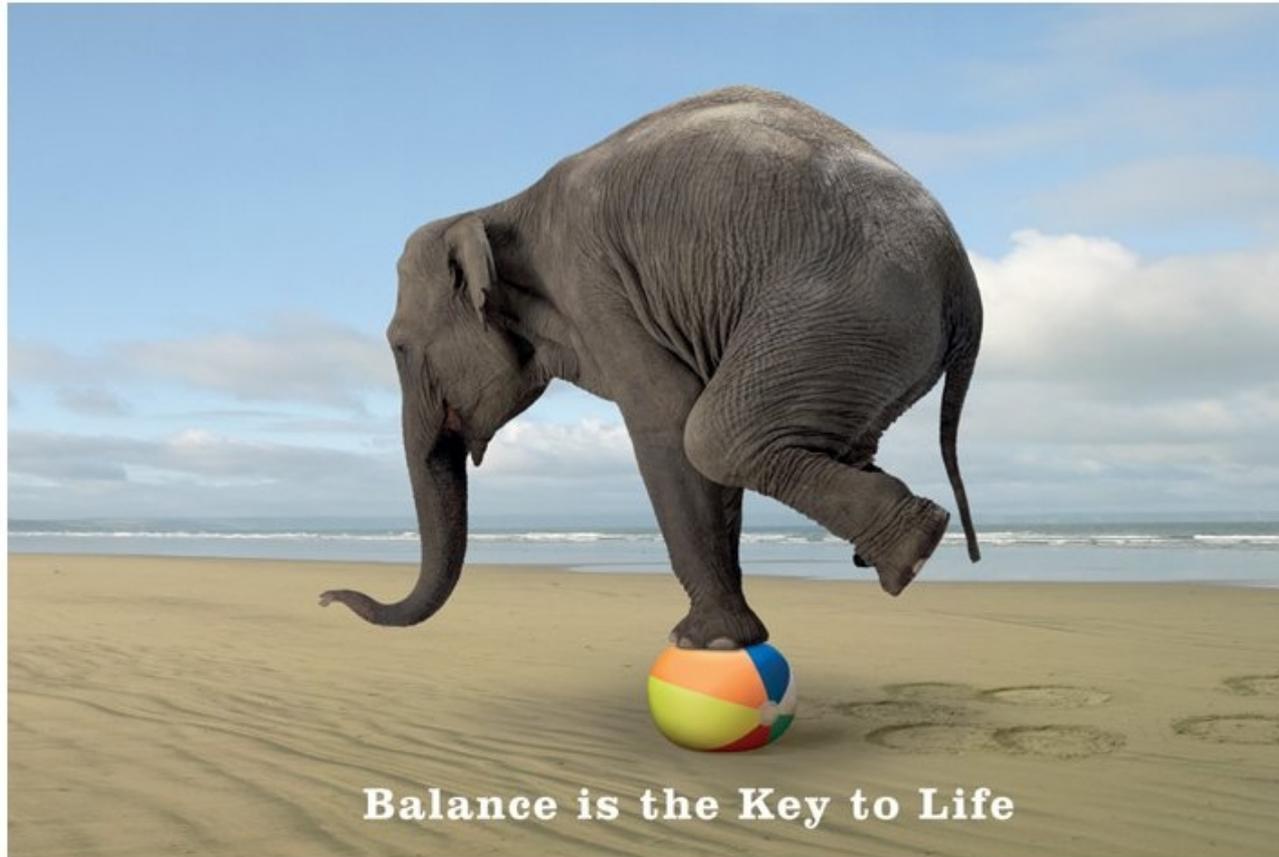
What are the limitations on control in gene regulatory networks?



What are different regulatory mechanisms optimized for?

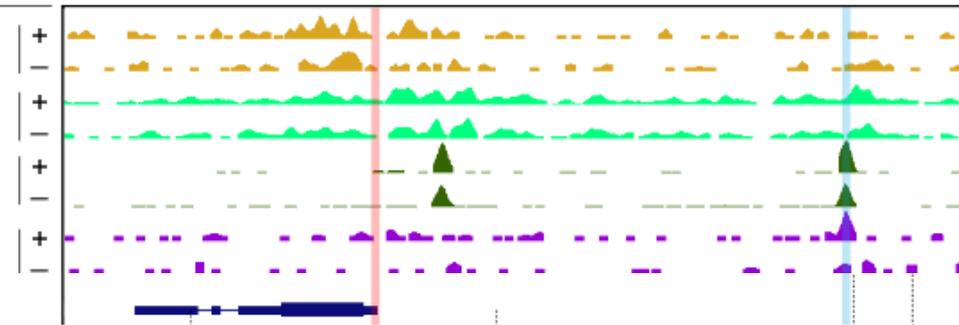


How robust is the system with respect to parametric perturbations?

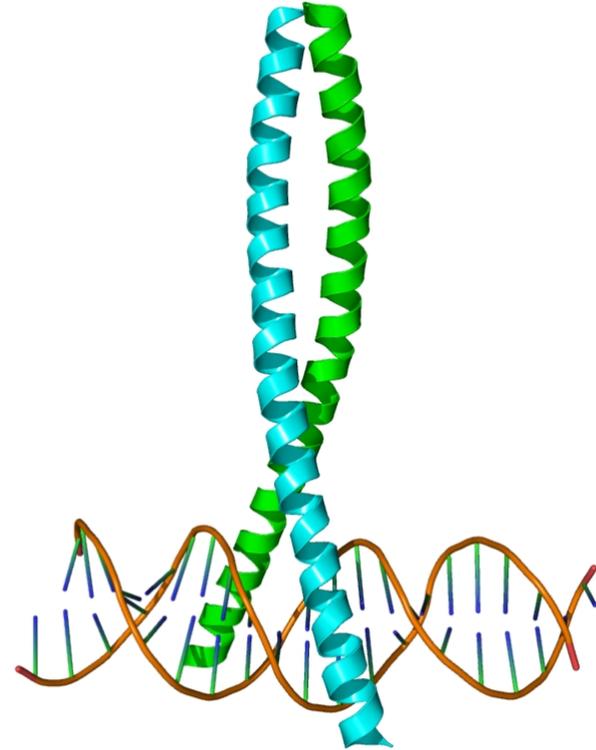


# What is the biophysical basis of TF binding?

- X-ray structures
- ChIP-Seq binding



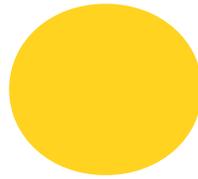
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# What is the impact of SNPs on TF binding?

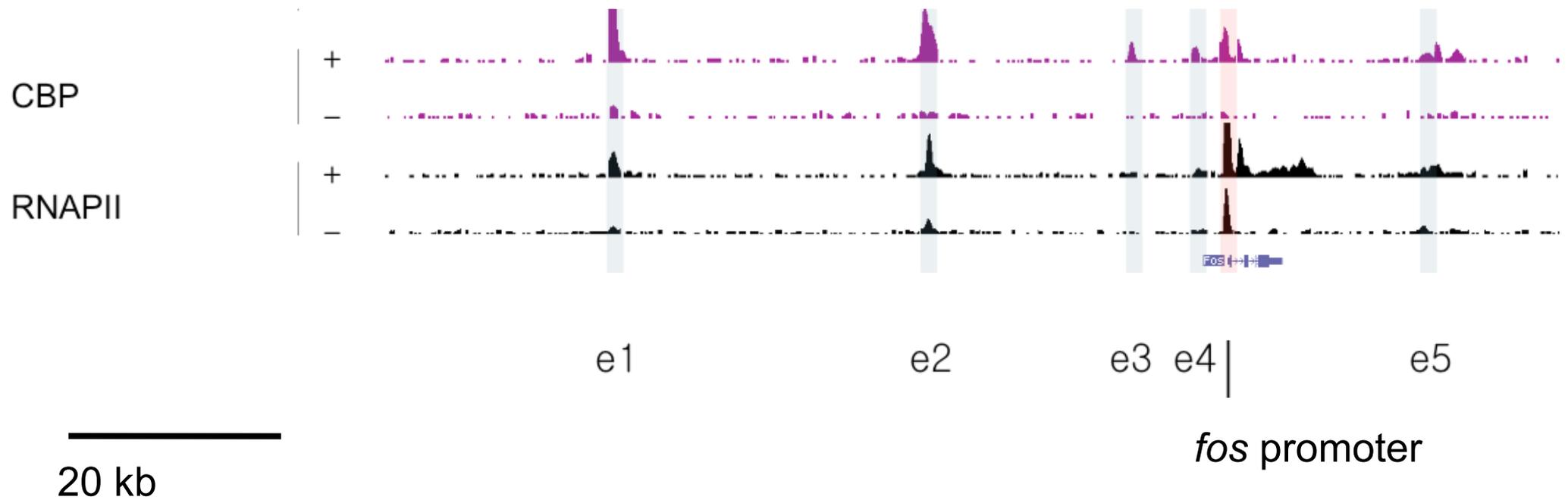
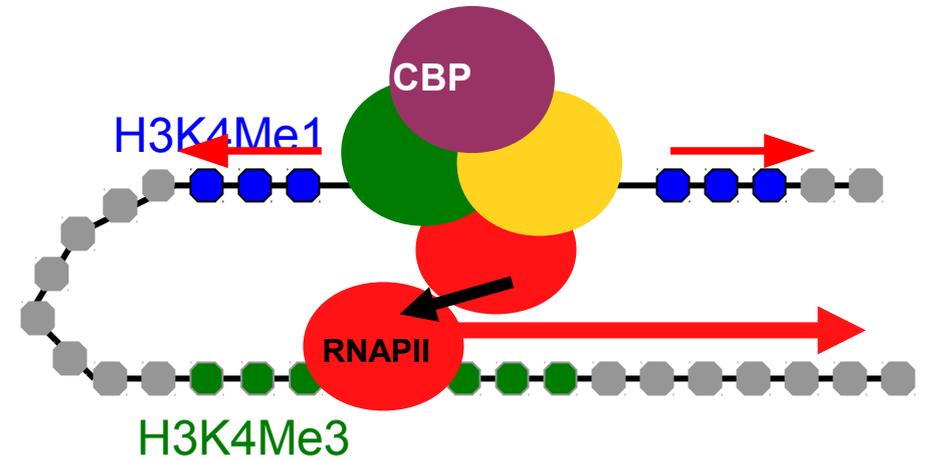


ACCTGACGTCAACGTTTAA



ACCTGACATCAACGTTTAA

# What is the biophysical basis of DNA looping?



# Acknowledgements

- Gabriel Kreiman
- Jesse Gray
- Tae-Kyung Kim
- Athar Malik
- Michael Greenberg

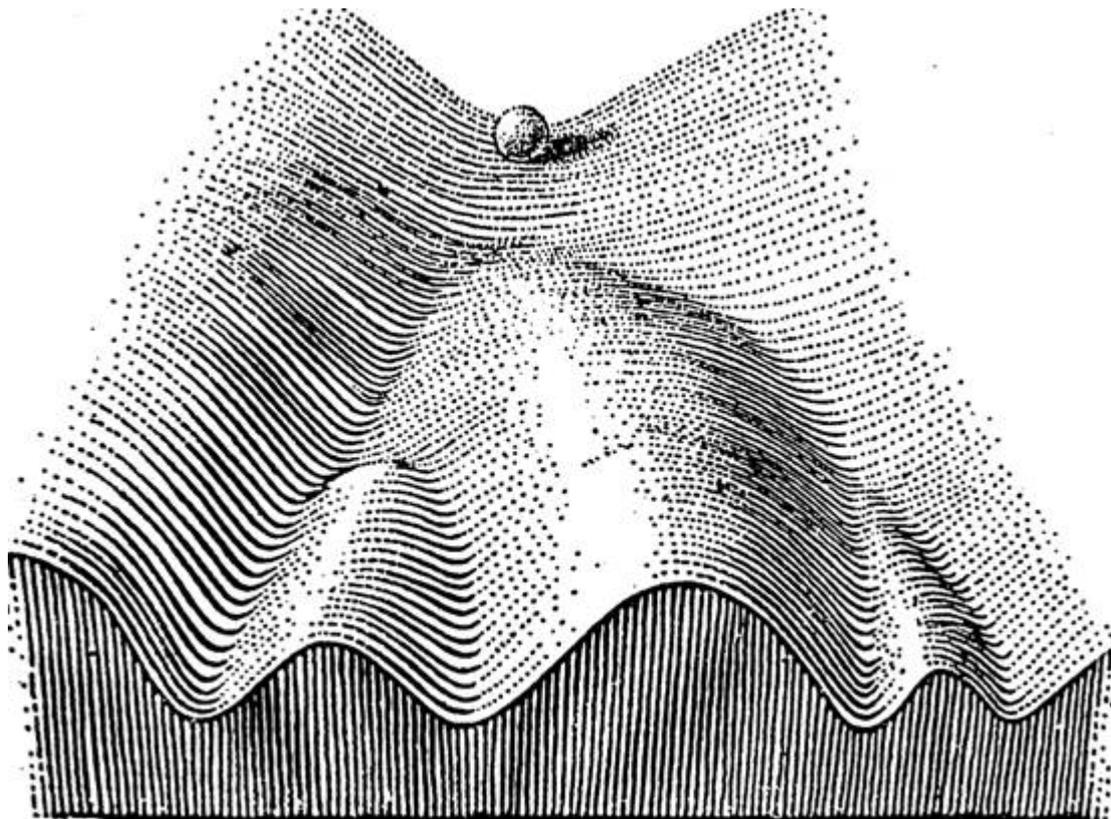
**Thank You**

■

?

# Stochastic models of gene expression

- Transitions between stable states



Waddington, 1953

# Enzymes piggyback on the polymerase

- What is the function of RNAPII at enhancers?

- Increase rate of RNAPII recruitment

- Possibly faster than diffusion limit

- Faster rise-time

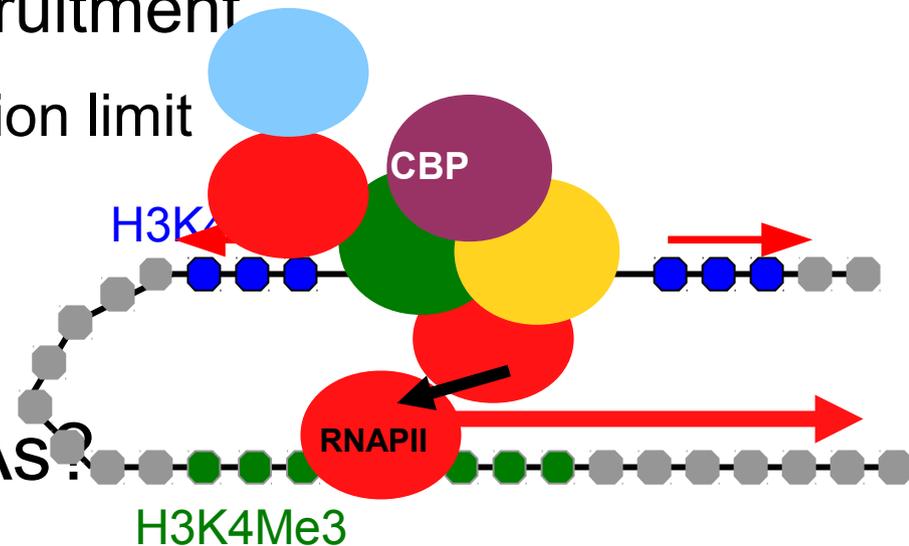
- Reduced noise

- What is the function of eRNAs?

- Noise

- Transcription establishes histone modifications

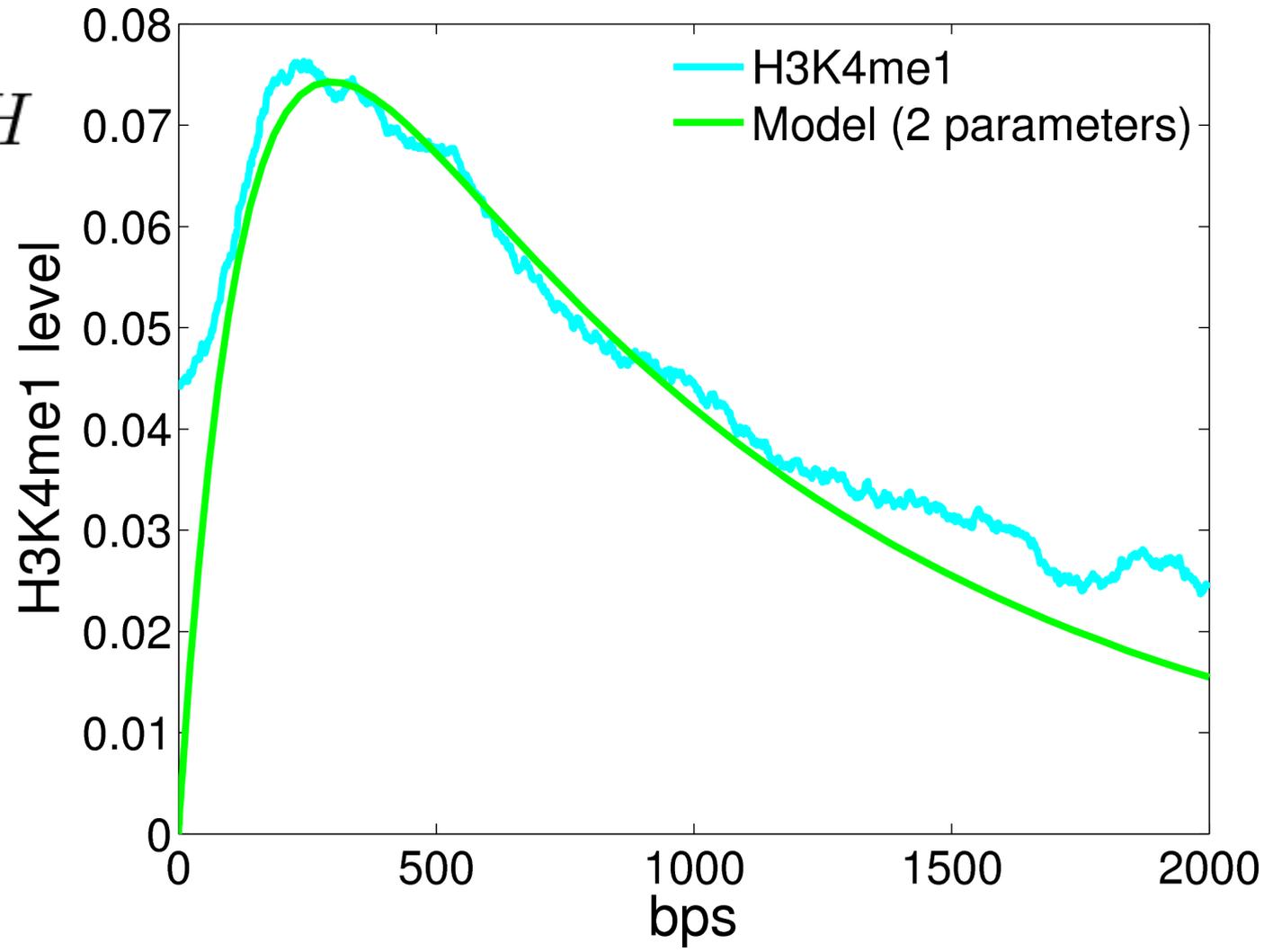
- Transcript has function



# Establishing H3K4me1 levels at enhancers

$$\frac{dP}{dx} = k - \lambda P$$

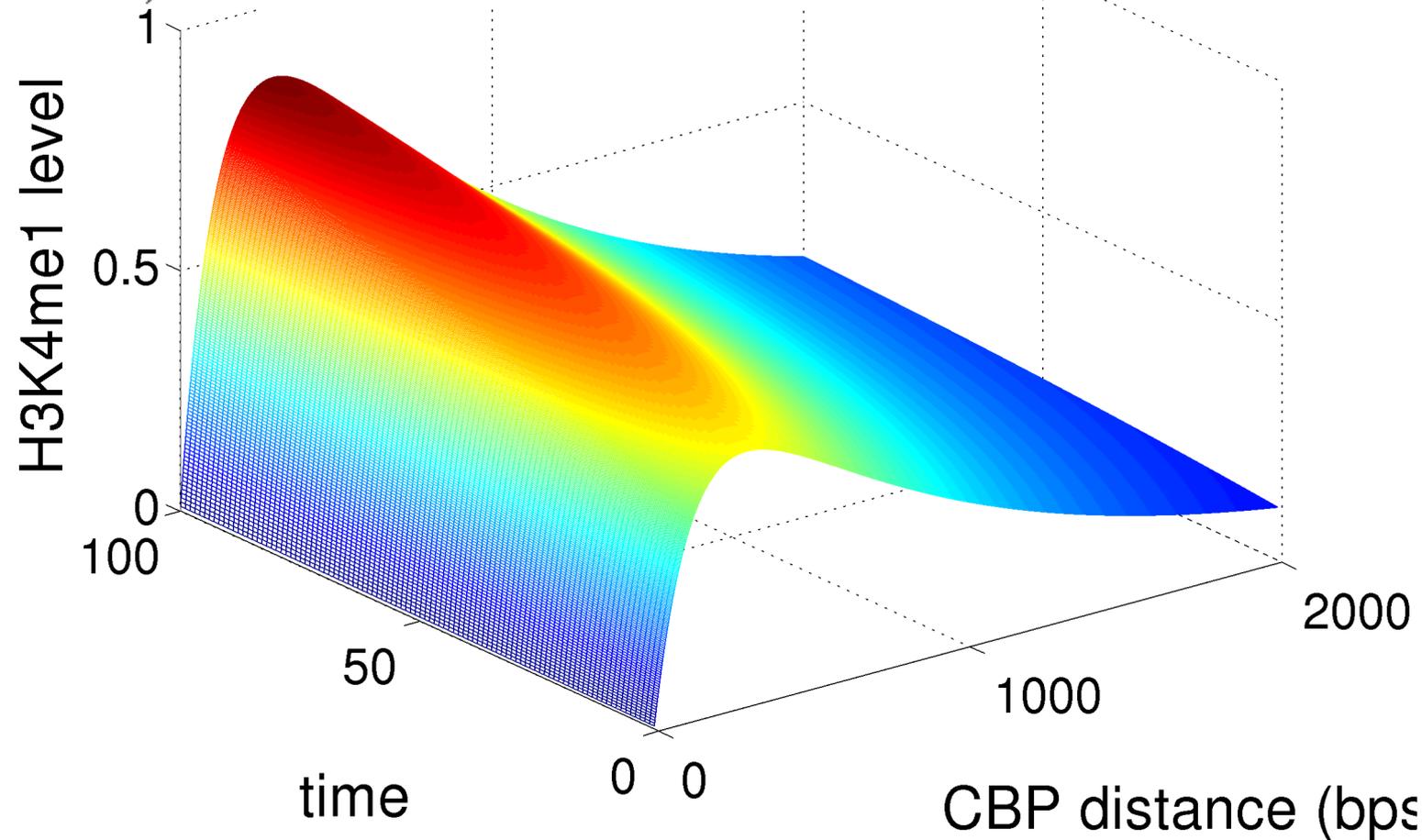
$$\frac{dH}{dx} = \kappa P(x) - \mu H$$



# A PDE for histone levels

$$\frac{\partial H}{\partial x} + \frac{\partial H}{\partial t} = \kappa P(x, t) - \mu_x H - \mu_t H$$

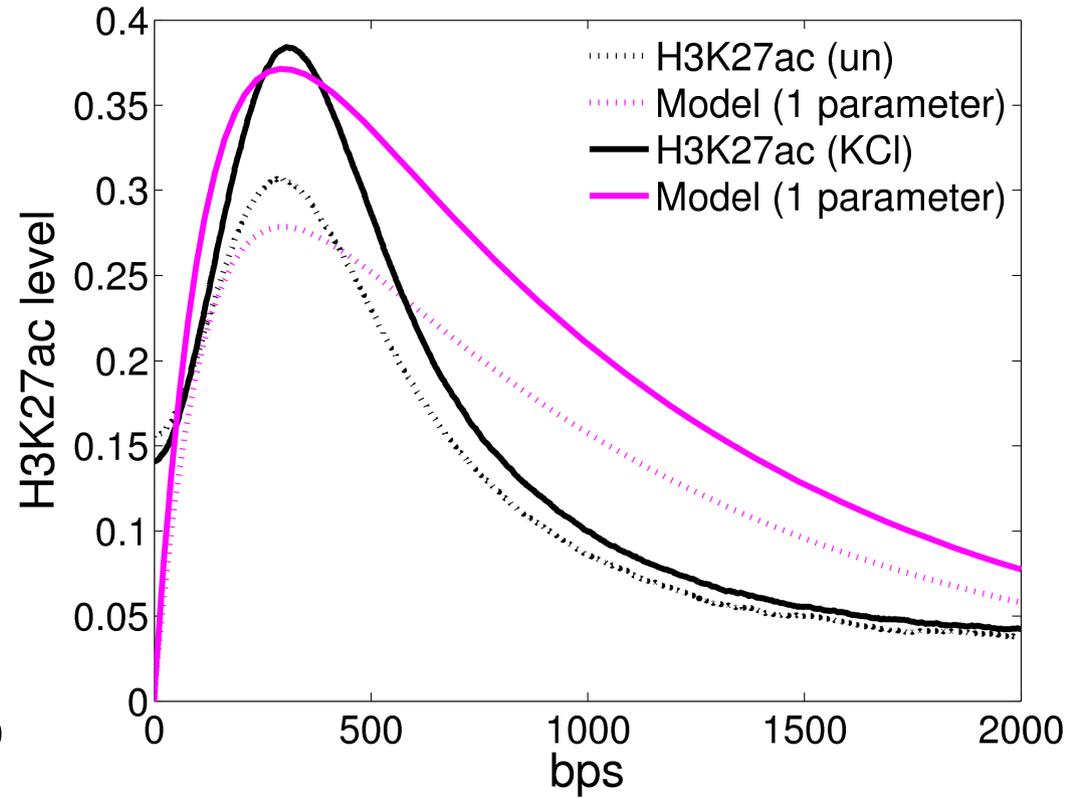
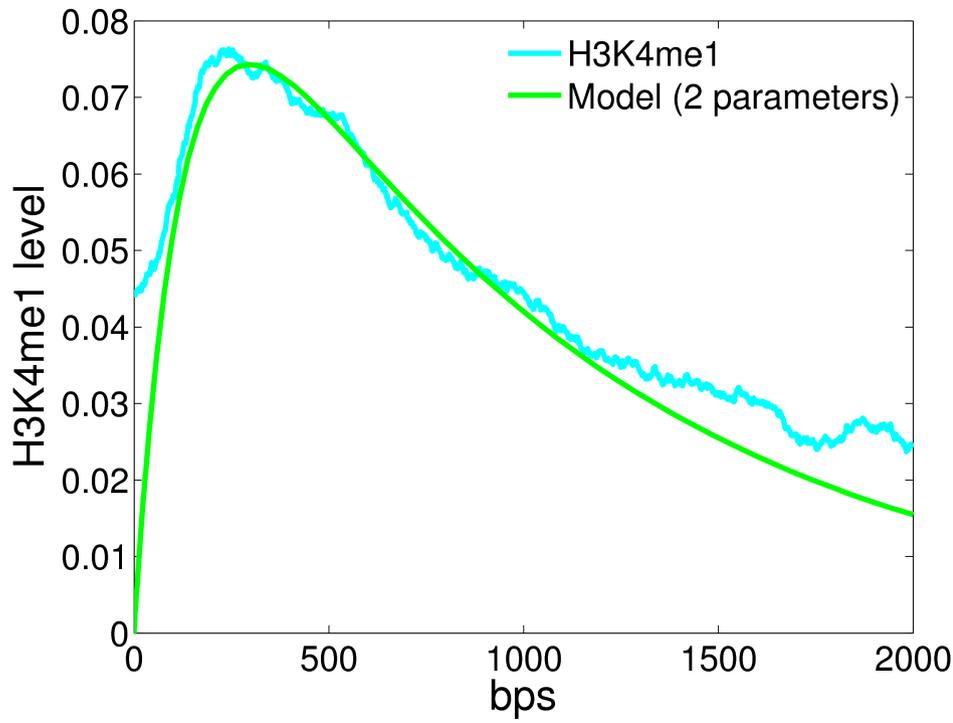
$$H(x, t) = \frac{\kappa \kappa}{\mu_x (\mu_x - \lambda)} (e^{-\lambda x} - e^{-\mu_x x}) \times e^{-\mu_t t}$$



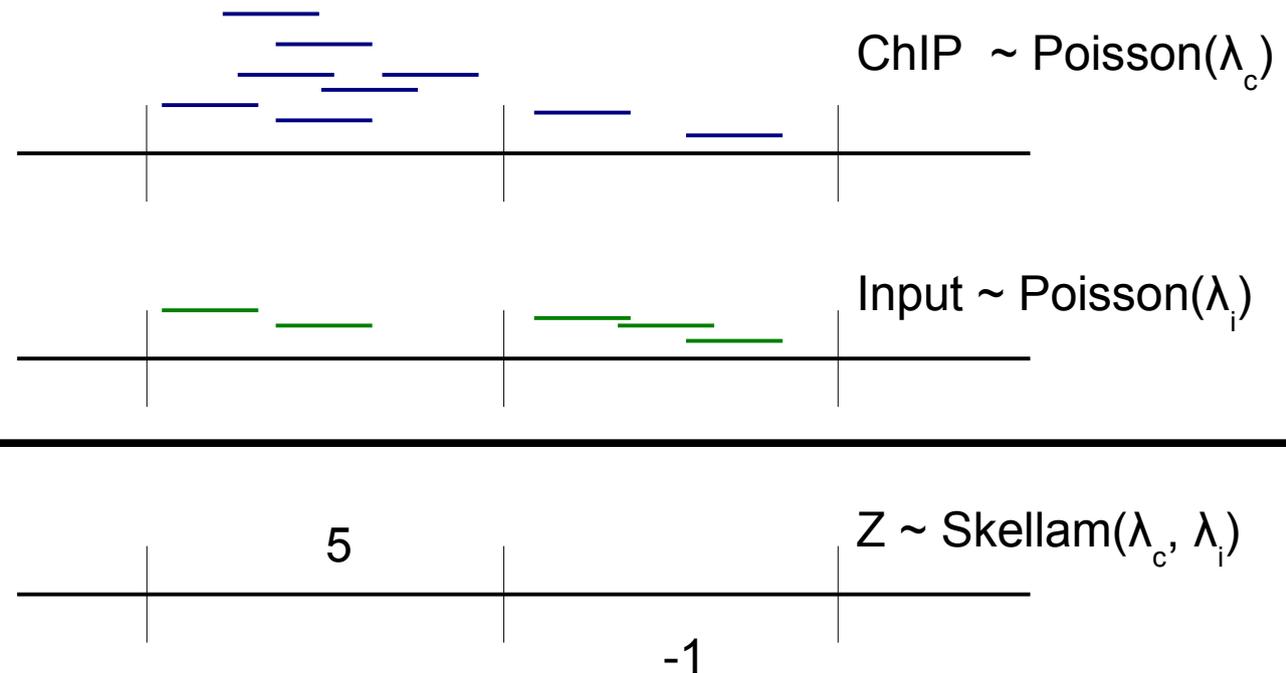
Histone methylation is not significantly changed, but histone acetylation is

$$\frac{\partial H}{\partial x} + \frac{\partial H}{\partial t} = \kappa P(x, t) - \mu_x H - \mu_t H$$

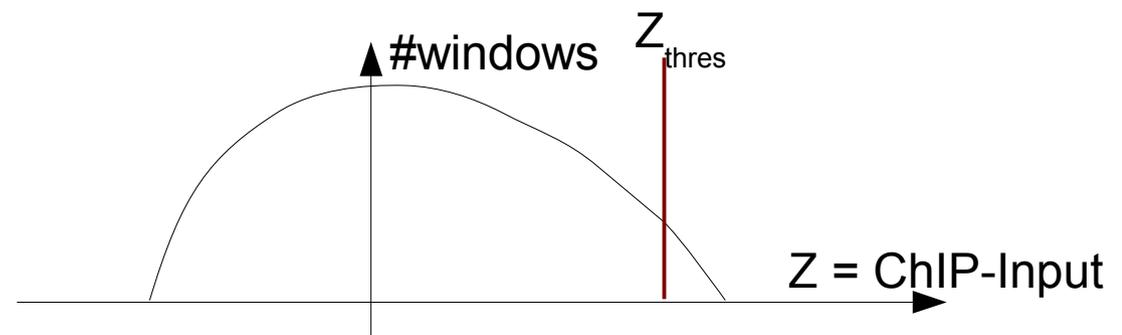
$$H(x, t) = \frac{\kappa \kappa}{\mu_x (\mu_x - \lambda)} (e^{-\lambda x} - e^{-\mu_x x}) \times e^{-\mu_t t}$$



# Identifying regions with larger than expected number of ChIP-Seq reads



- False Detection Rate (FDR) determine threshold



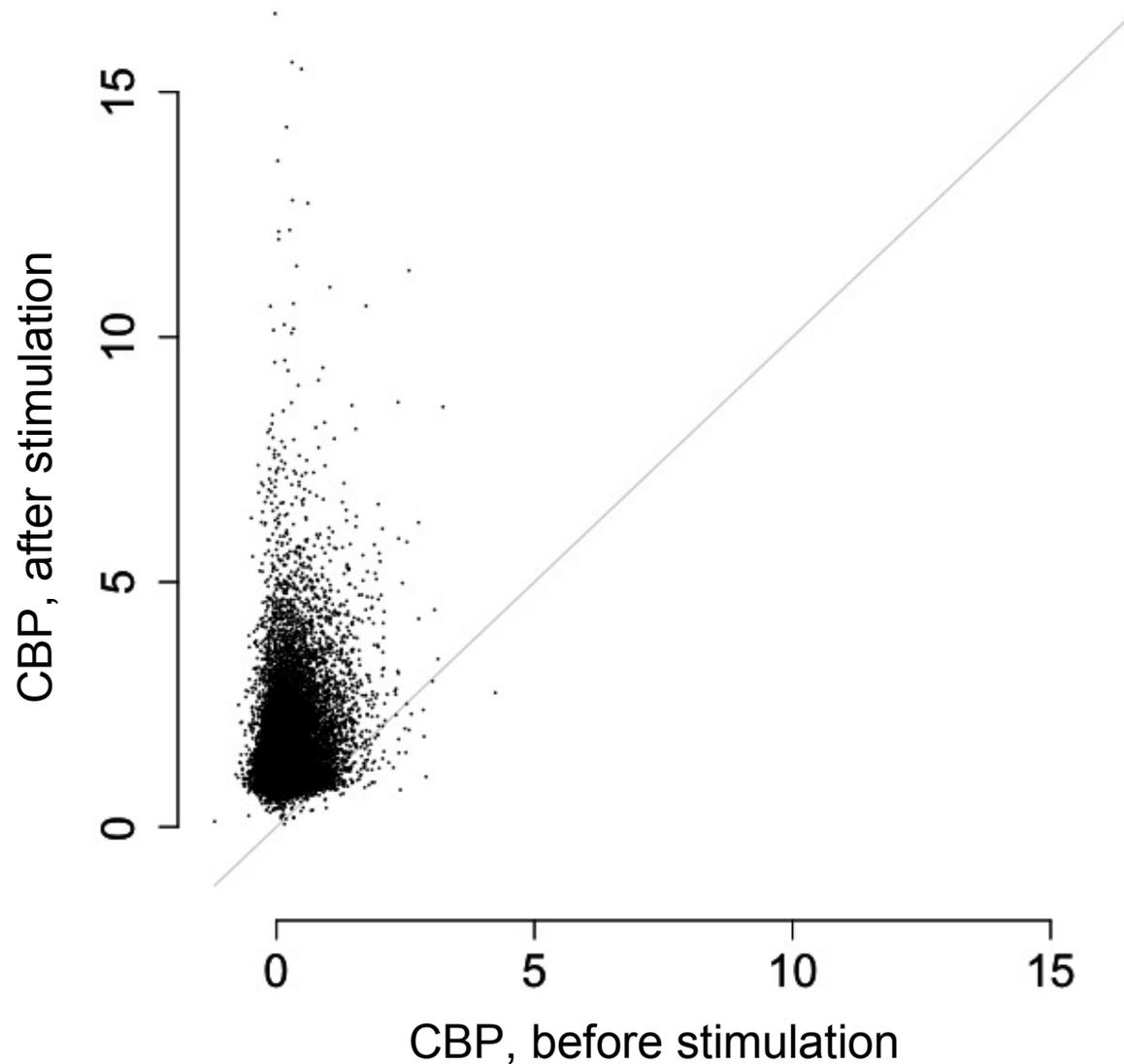
# Use False Detection Ratio (FDR) to correct for multiple hypotheses

- $Z_i = \text{\#ChIP reads} - \text{\#input reads in window } i$
- $\sim 1 \text{ read}/100 \text{ bp}$ 
  - Assume  $\text{\#reads in window } P(k) = \lambda^k \exp(-\lambda)/k!$ 
    - Difference between two Poisson random variables
    - $Z_i \sim \text{Skellam}(z, \lambda_1, \lambda_2)$

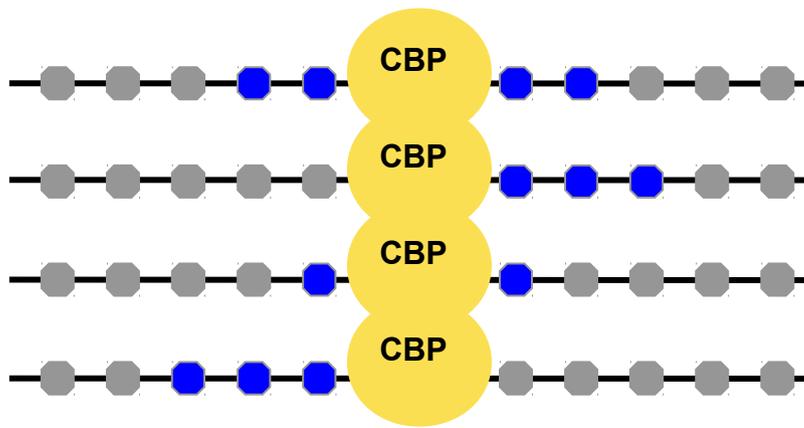
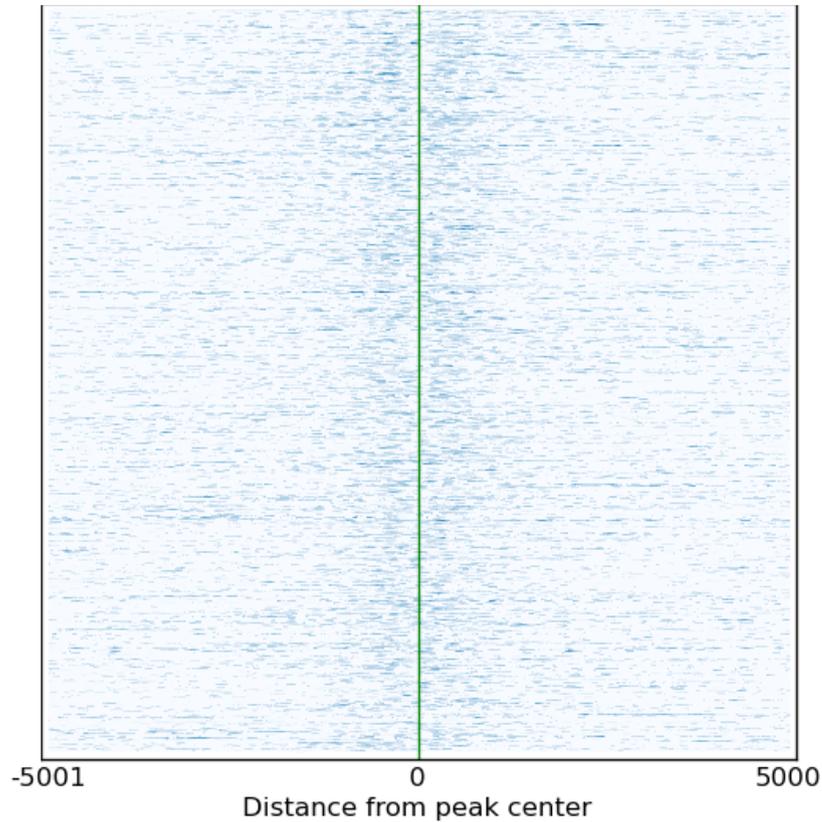
$$p(x) = e^{-(\lambda_1 + \lambda_2)} (\lambda_1 / \lambda_2)^{x/2} I_x(2 \sqrt{\lambda_1 \lambda_2})$$

- Millions of windows need to be tested
  - FDR - expected fraction of false positives

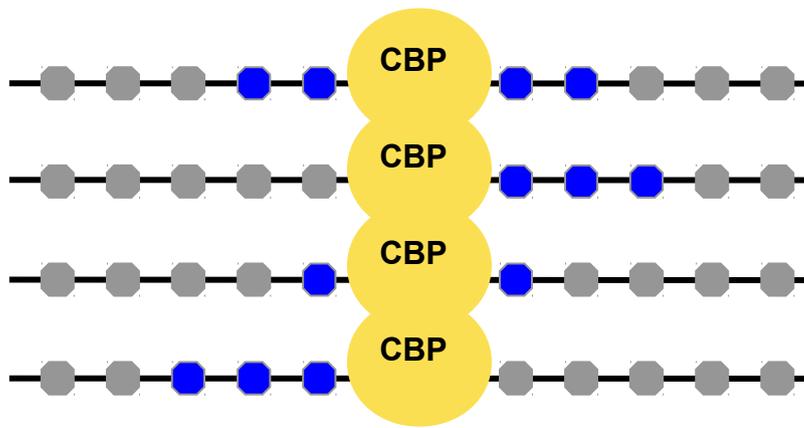
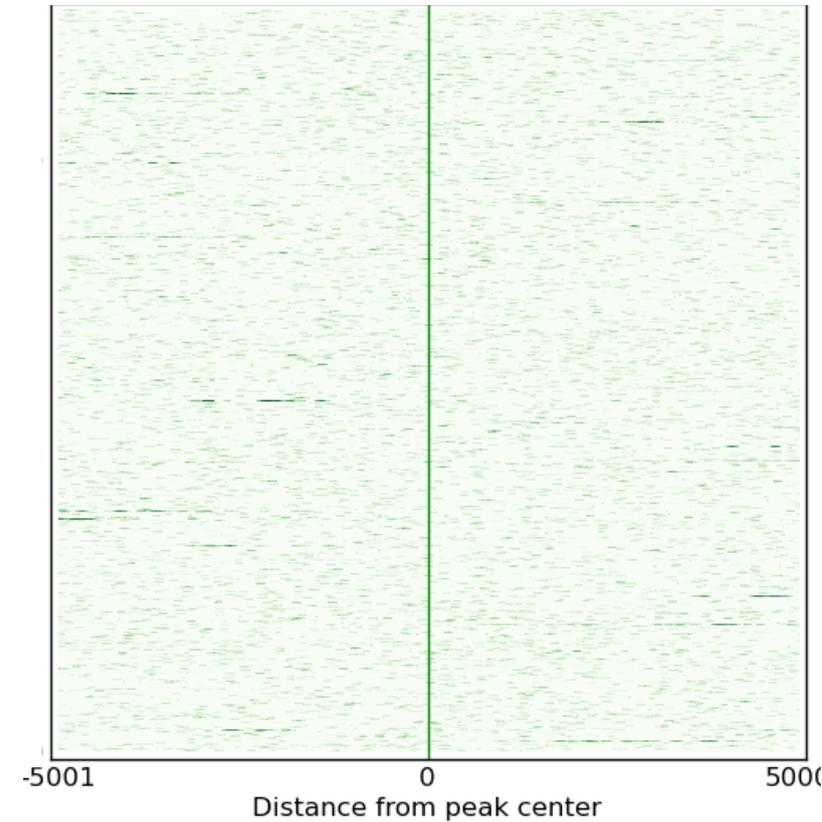
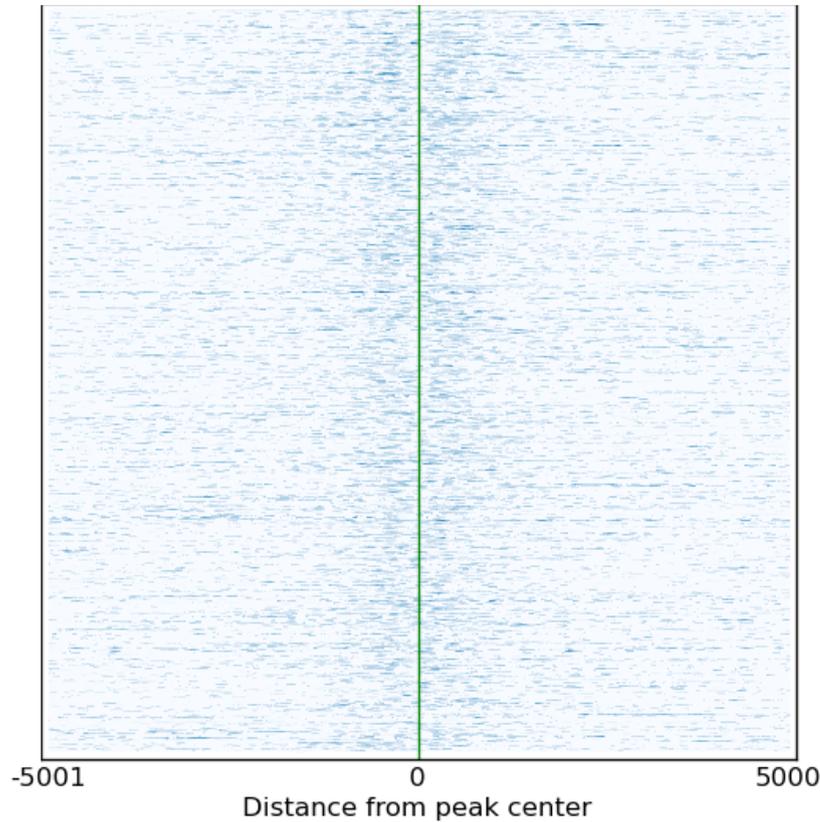
CBP binds in an activity regulated manner to  
~28,000 sites throughout the genome



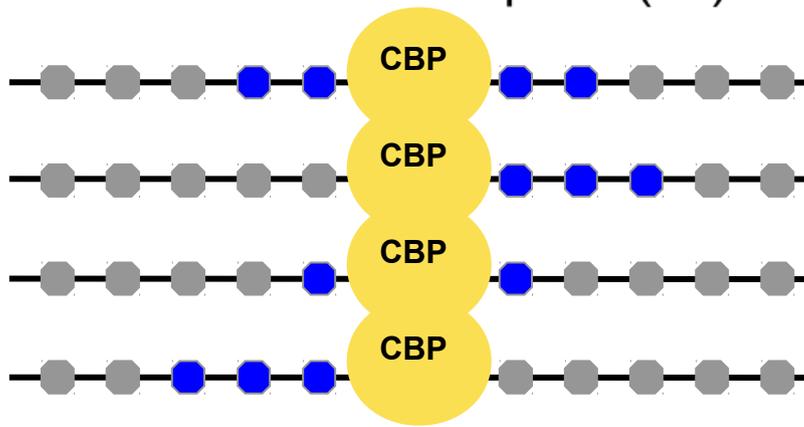
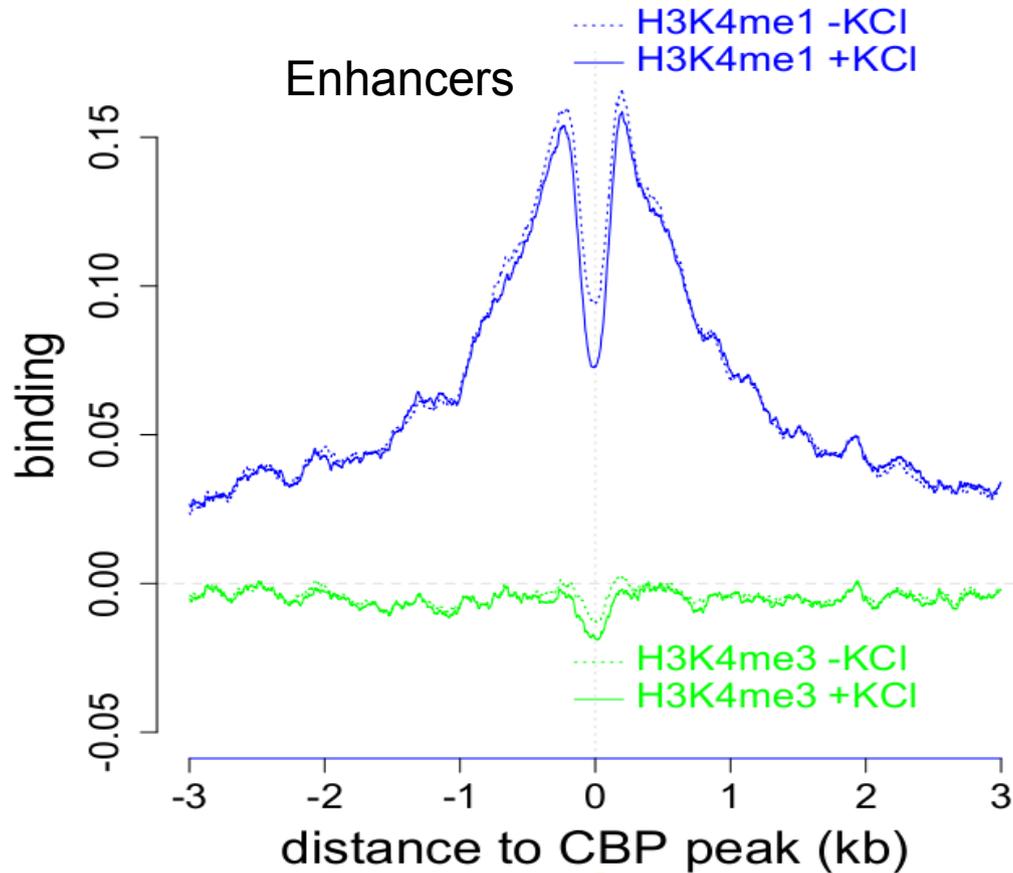
# Aligning CBP peaks to calculate H3K4me1 binding profiles



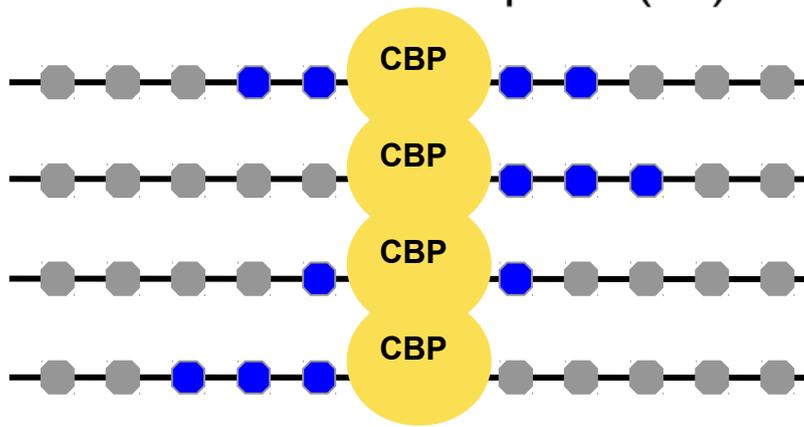
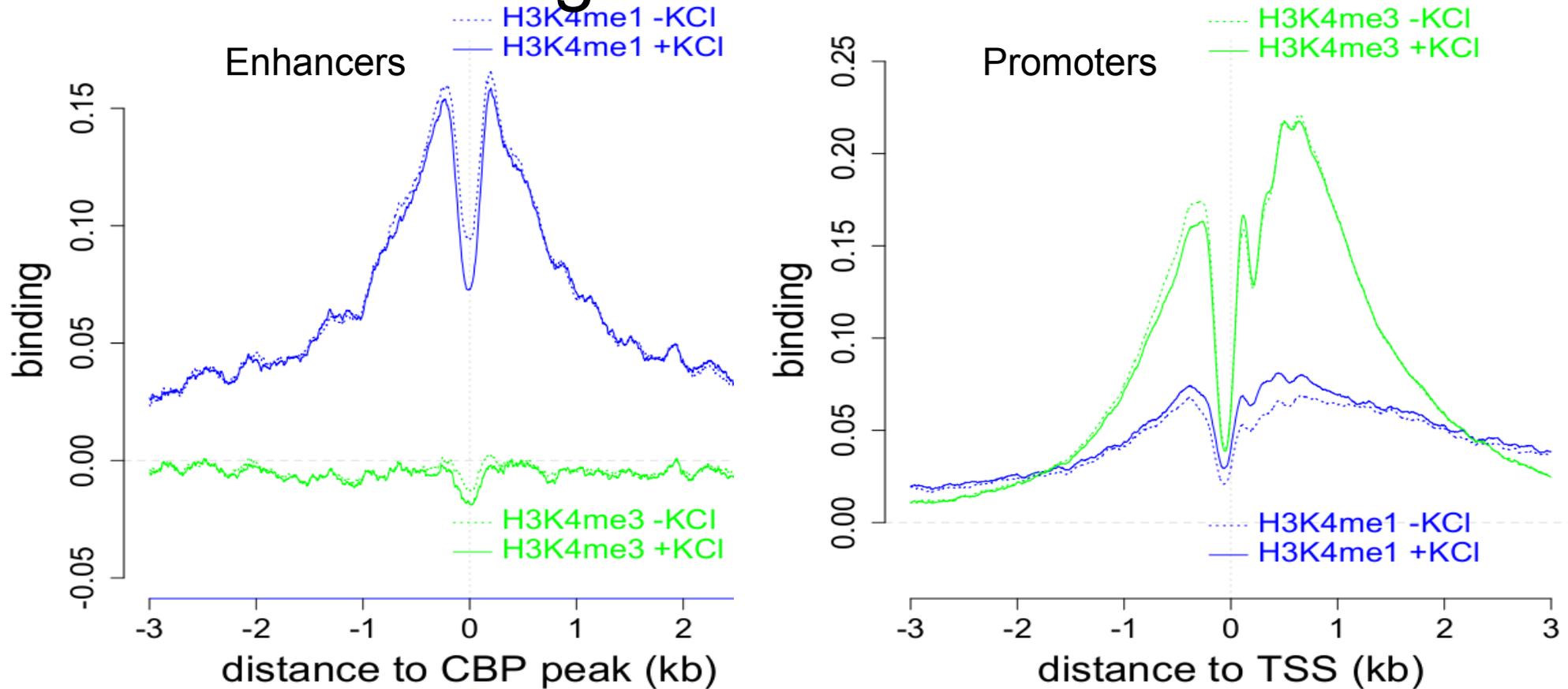
# Aligning CBP peaks to calculate H3K4me1 and H3K4me3 binding profiles



# Enhancers have high levels of H3K4me1 and low levels of H3K4me3

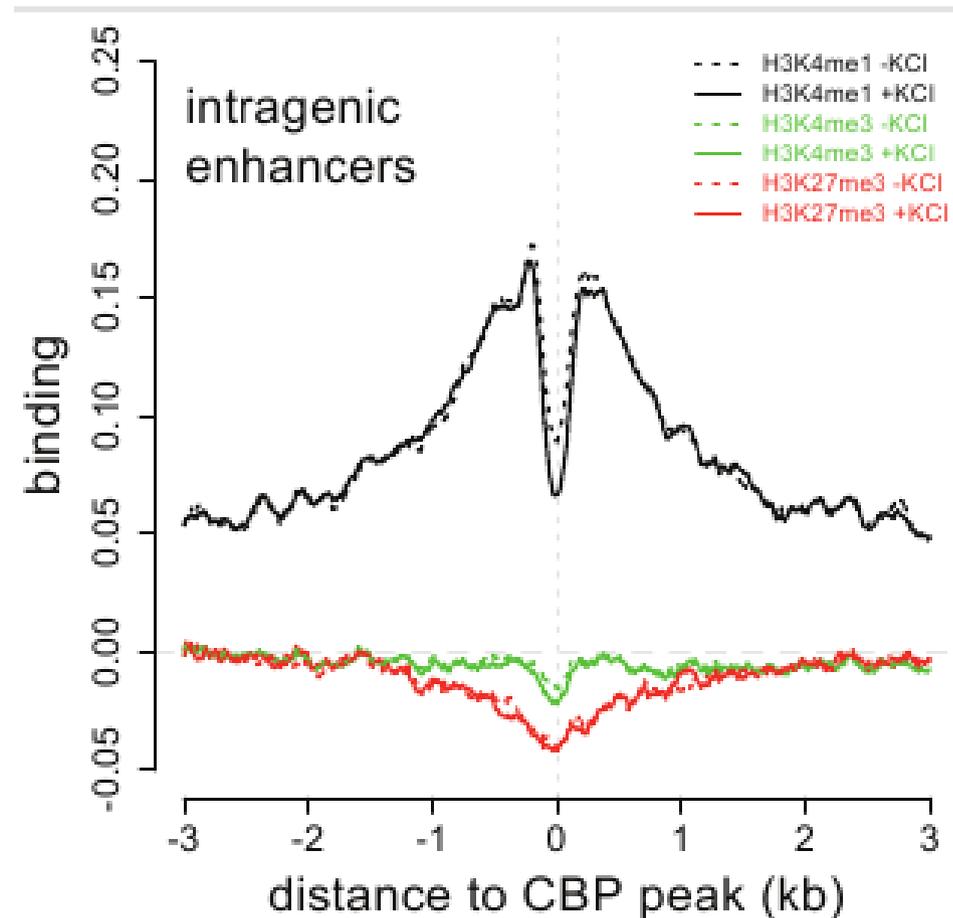


# Transcription start sites have high levels of H3K4me1 and high levels of H3K4me3



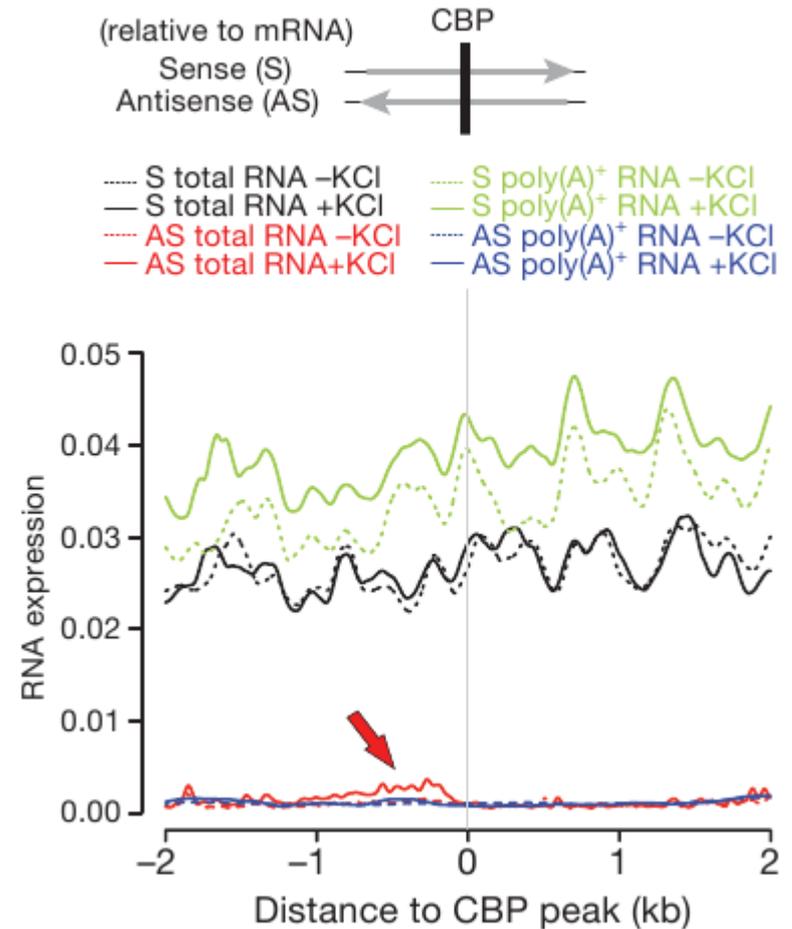
# Intragenic enhancers

- ~7,000 enhancers overlapping introns
  - H3K4me1, but no H3K4me3



# Intragenic enhancers are also transcribed

- ~7,000 enhancers overlapping introns
  - No signal detectable on sense strand
  - Significant anti-sense transcription



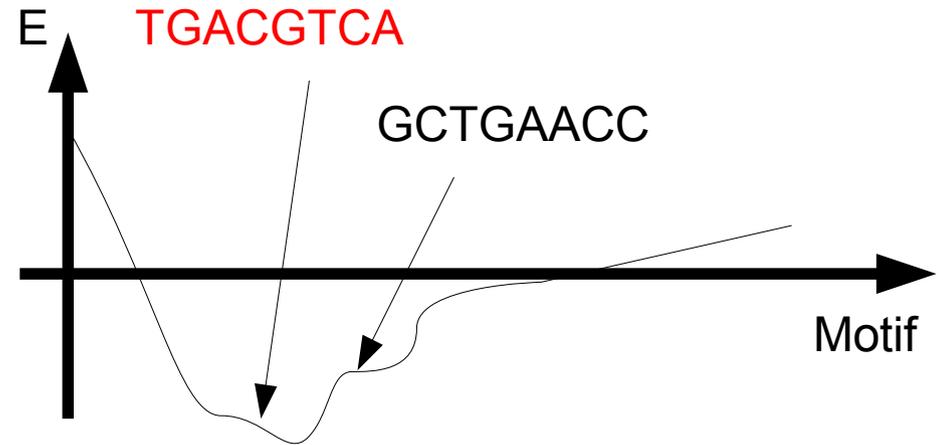
# Can the read count be predicted from sequence motifs?

$$R = f(E(\theta)) + \xi$$

#reads

Binding energy

noise



# Assume transfer function maximizes mutual information

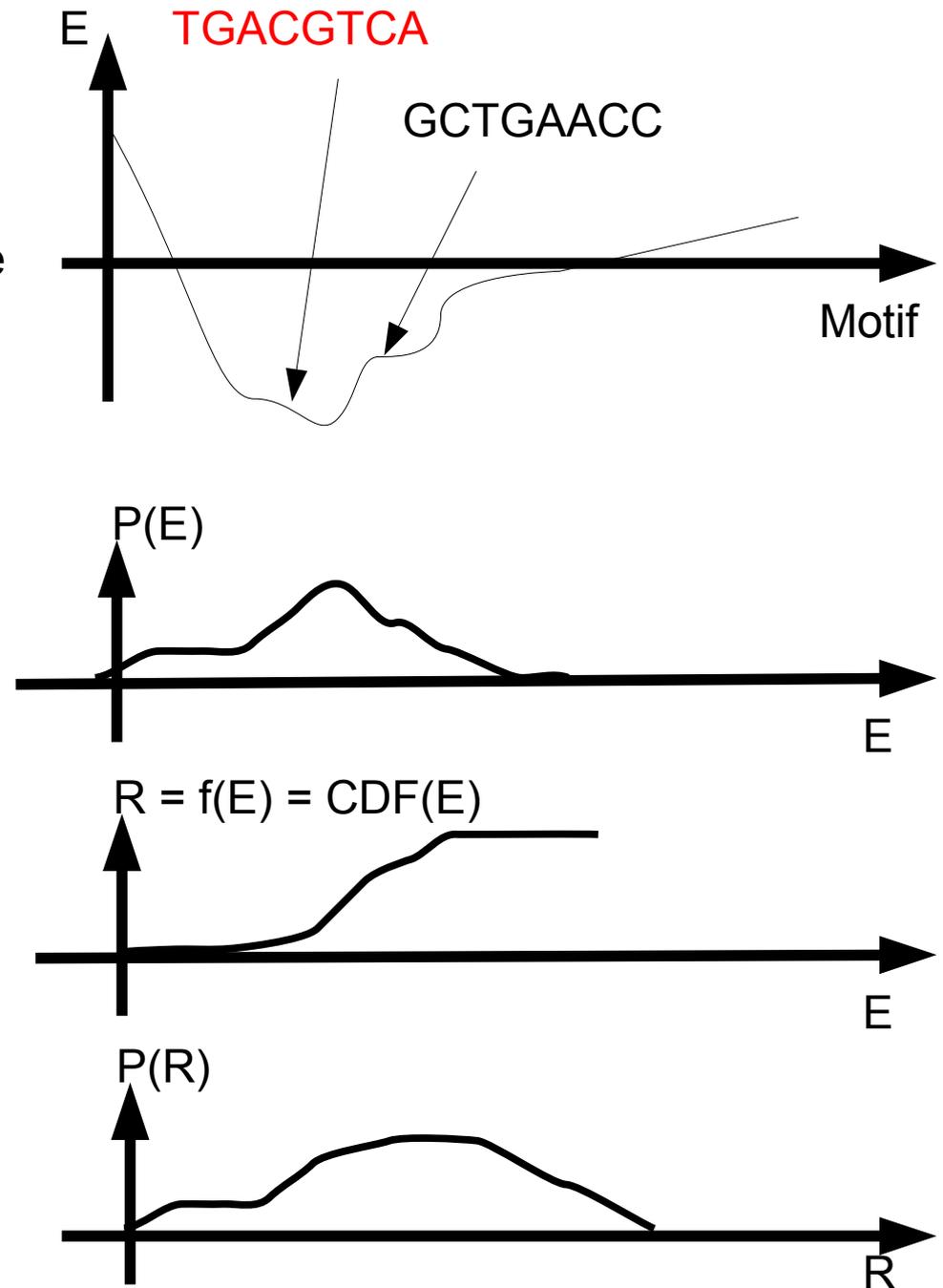
$$R = f(E(\theta)) + \xi$$

#reads  $\uparrow$   $R$

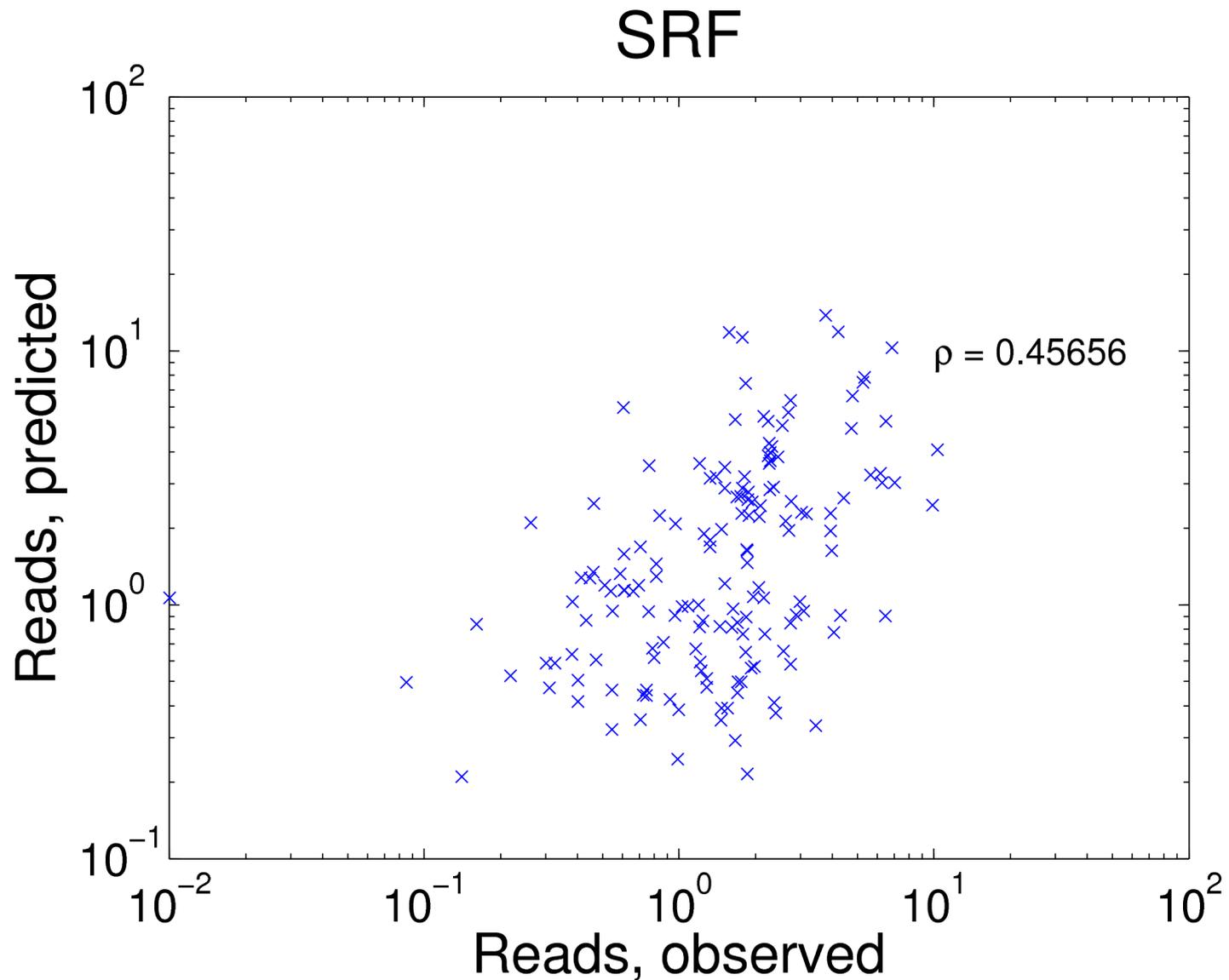
Binding energy (from JASPAR)  $\uparrow$   $E(\theta)$

noise  $\uparrow$   $\xi$

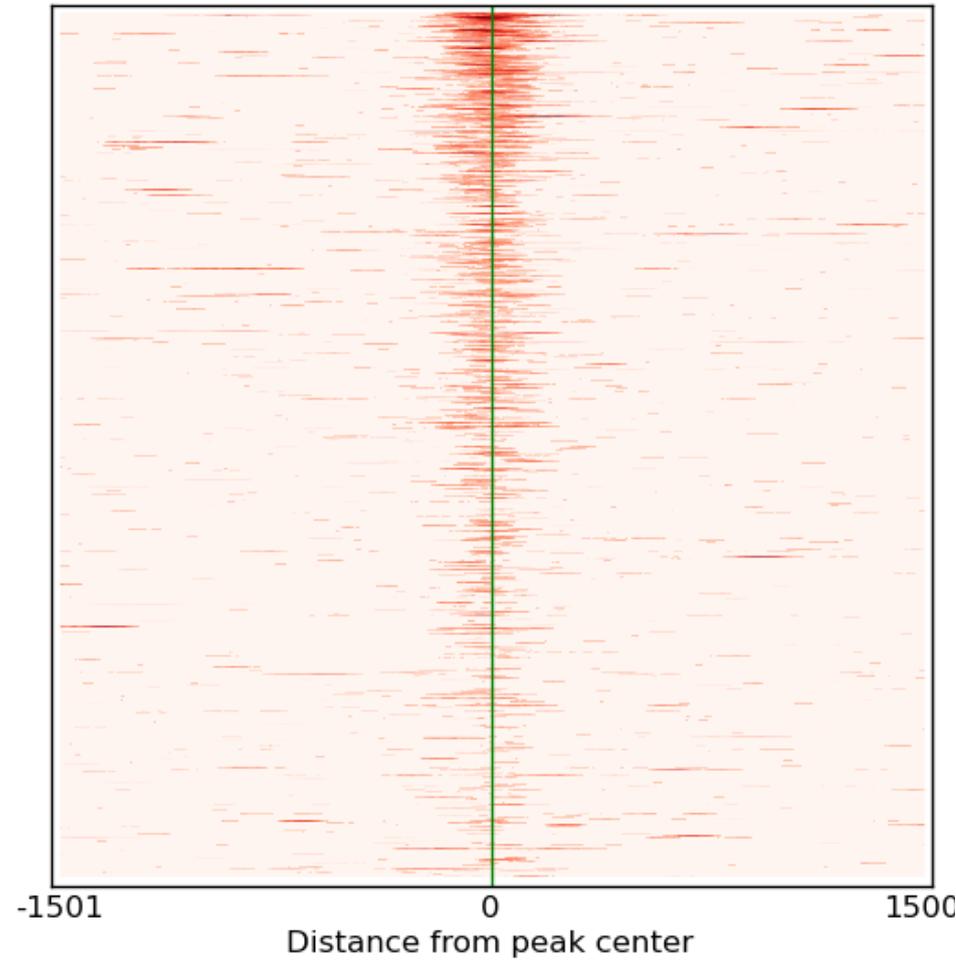
- $f$  monotone
- $\max I(R; E)$
- Noise small and gaussian



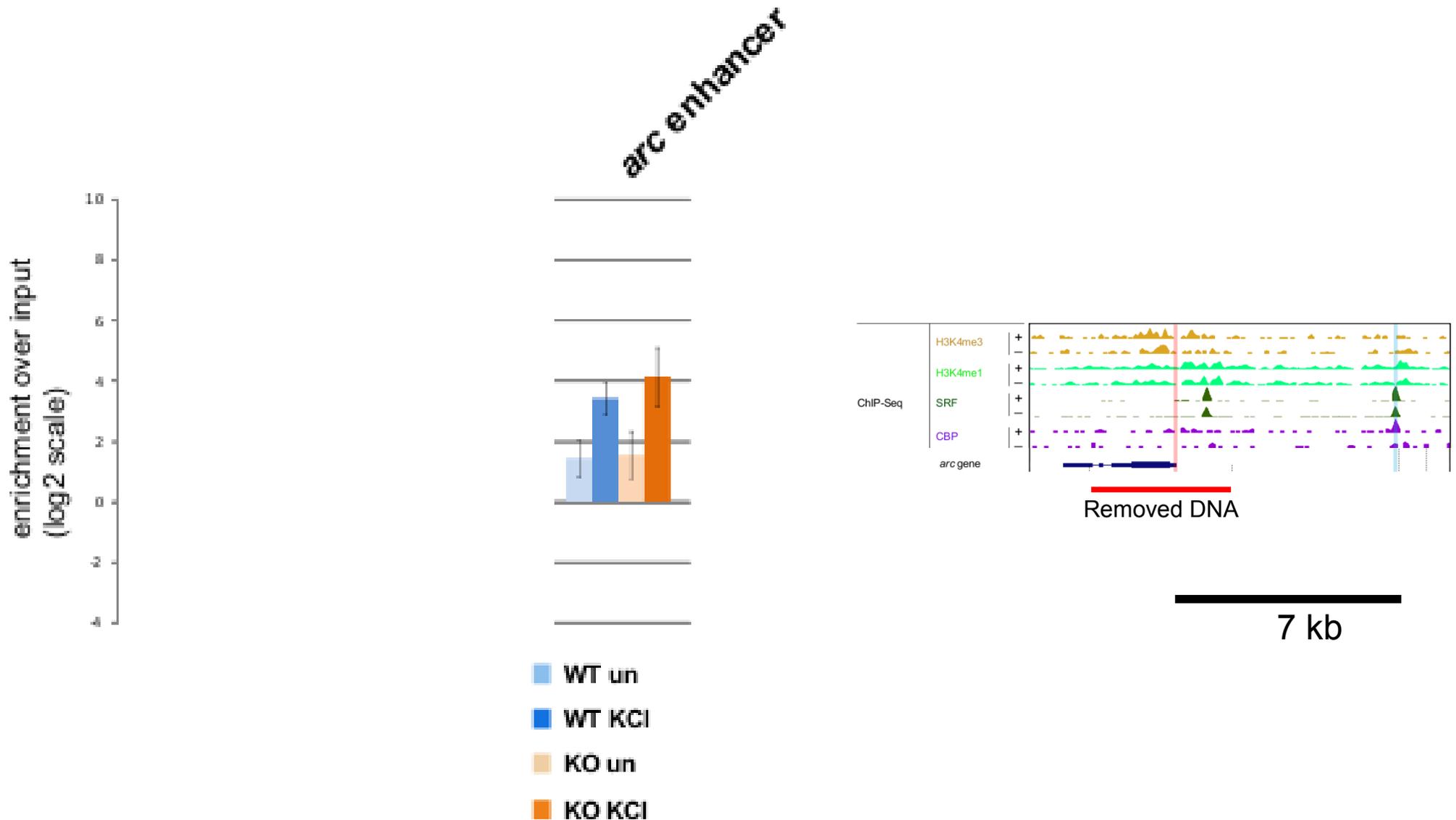
# Number of reads can be predicted by binding energy



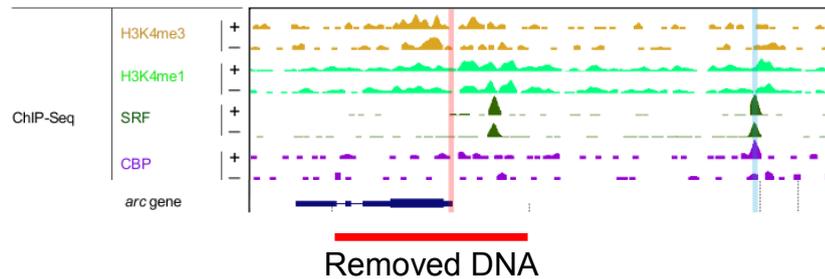
# RNAPII binds at activity-dependent enhancers



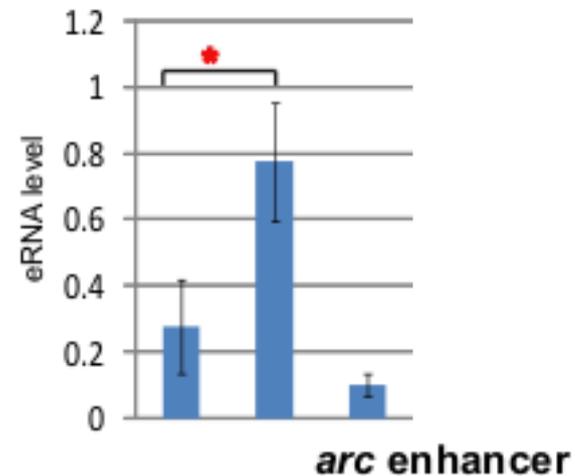
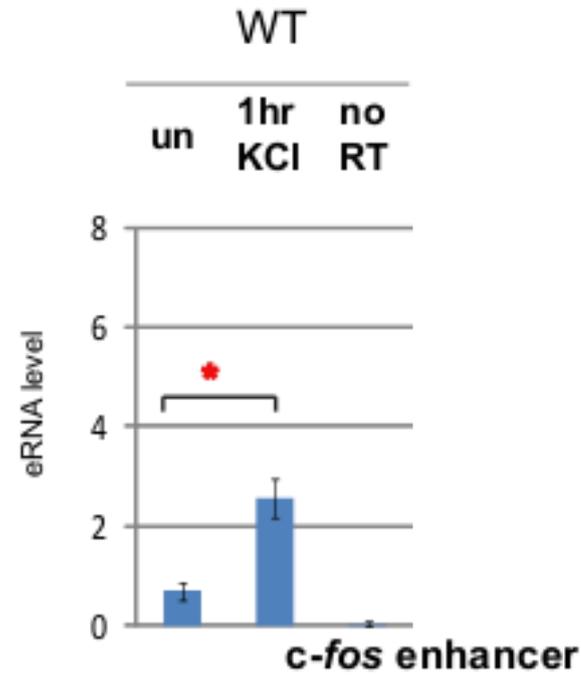
# RNAPII levels are unchanged at the enhancer in the mutant before and after KCl



# Transcription at the Fos and Arc enhancers



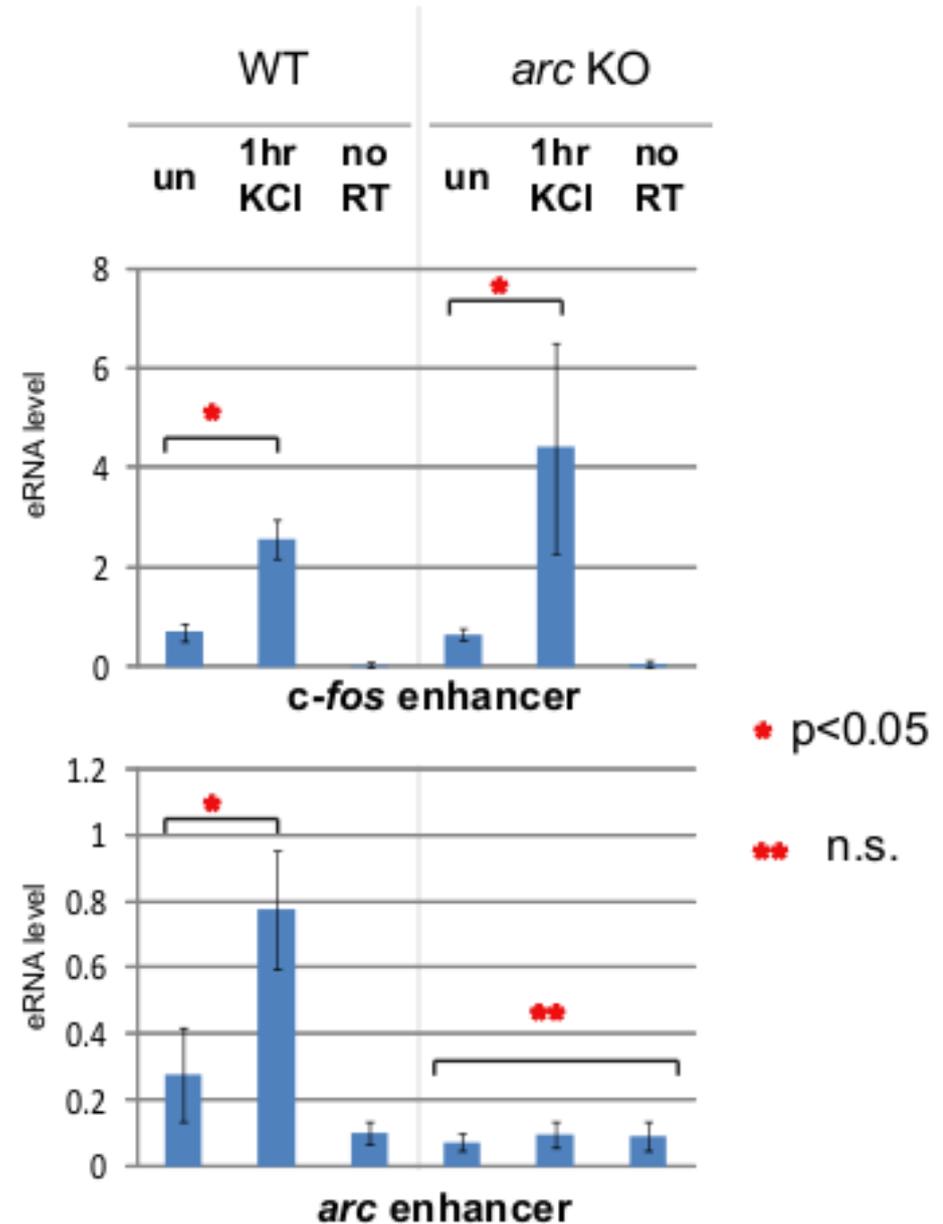
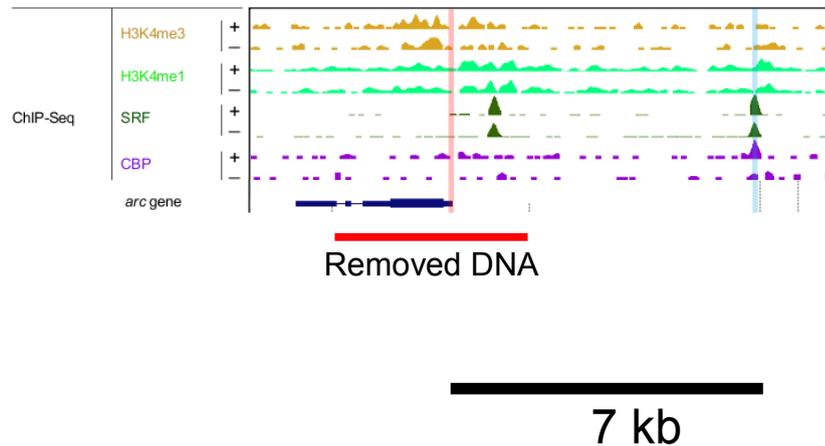
7 kb



\* p < 0.05

\*\* n.s.

# No transcription at Arc enhancer in mutant



# Estimating the production rate of eRNAs

$$\frac{dE}{dt} = kN - \frac{E}{\tau_E}$$

$$k = \frac{E^*}{N\tau_E} \sim \frac{10^3}{10^4 \times 10^{-1}\text{h}} = 1\text{h}^{-1}$$

$$\frac{\text{Variance strong promoter}}{\text{Variance weak promoter with enhancers}} = \frac{\text{Var}[(1 + Nc)k]}{\text{Var}[k] + N\text{Var}[ck]} = \frac{(1 + Nc)^2\text{Var}[k]}{(1 + Nc^2)\text{Var}[k]} \sim N$$

# Parameters for the eRNA fit

$$\lambda = \frac{k_{drop} \text{ s}^{-1}}{k_{elong} \text{ bp}^{-1} \text{ s}^{-1}} \sim \frac{2 \times 10^{-2}}{20} \text{ bp}^{-1} = 10^{-3} \text{ bp}^{-1}$$

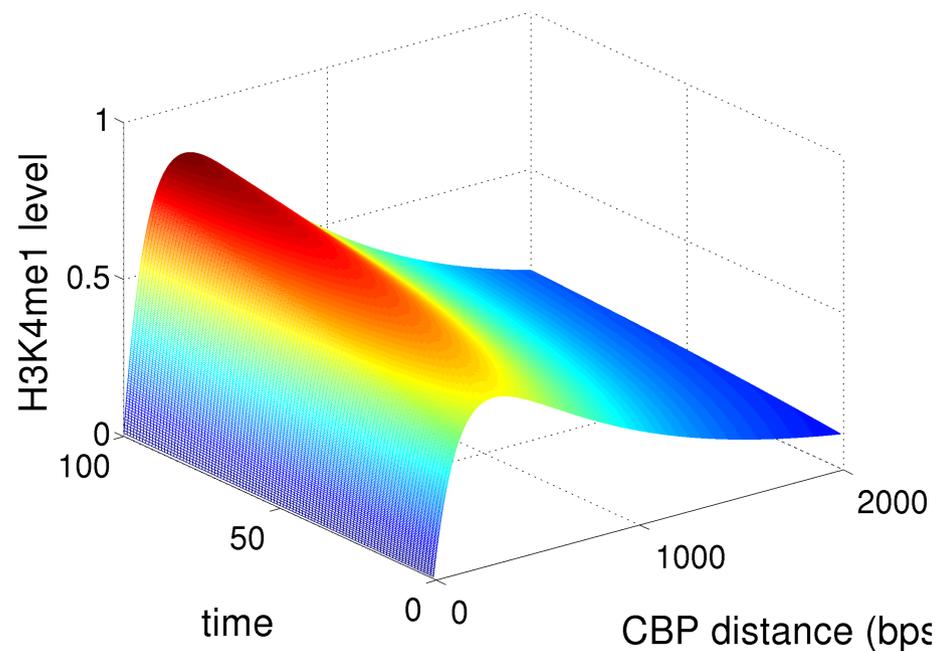
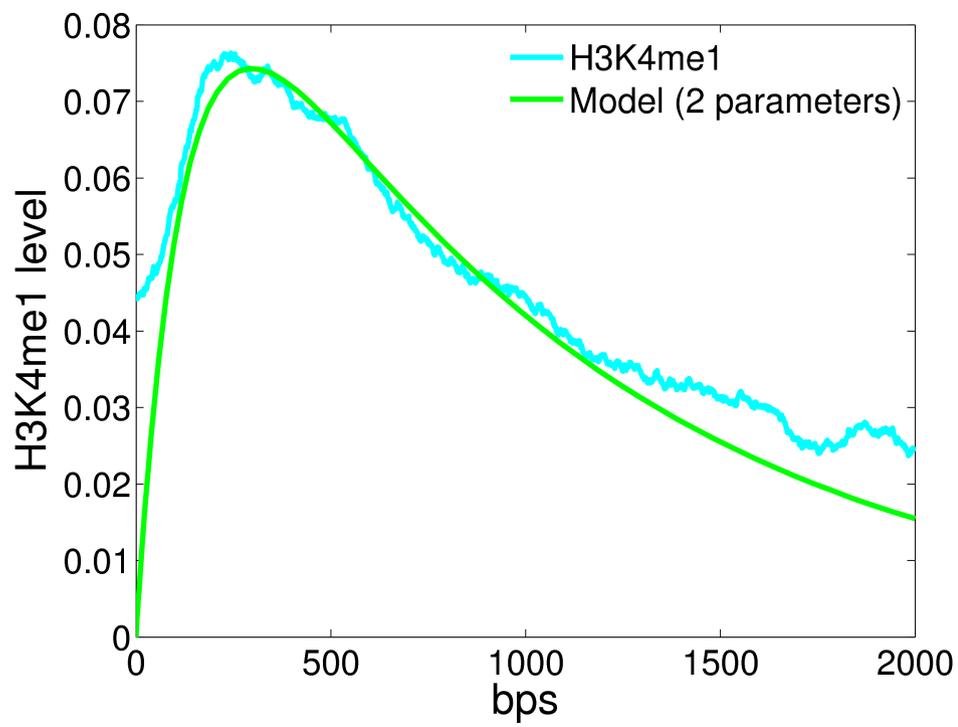
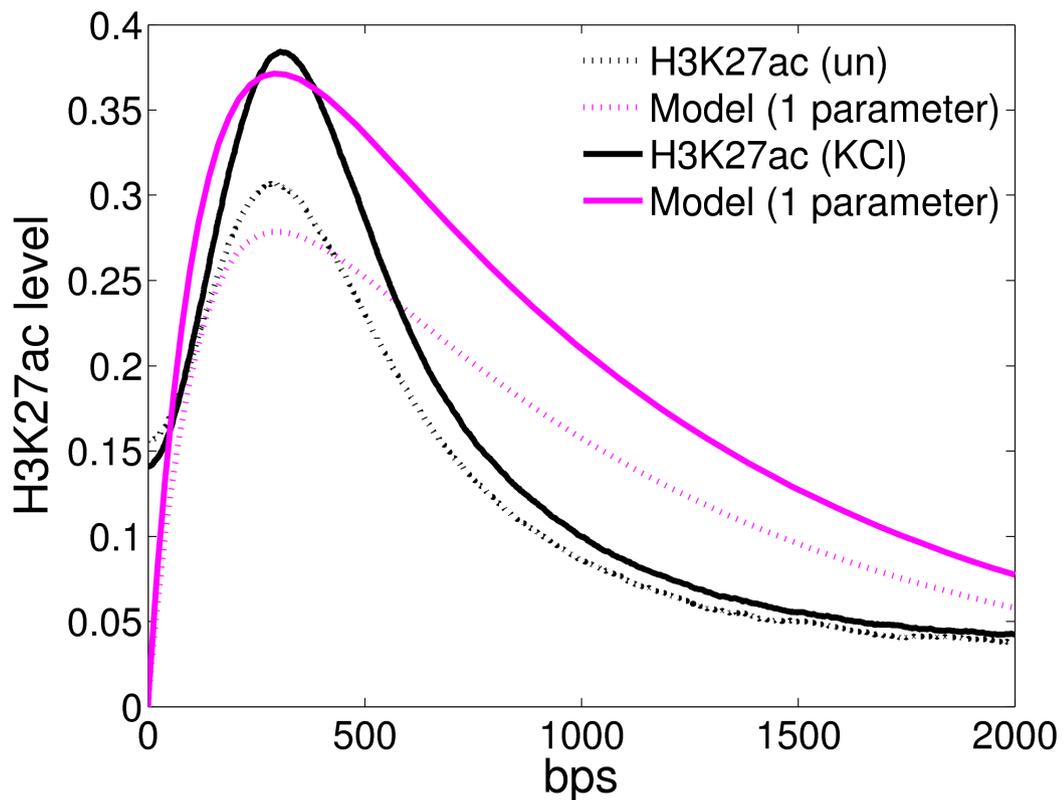
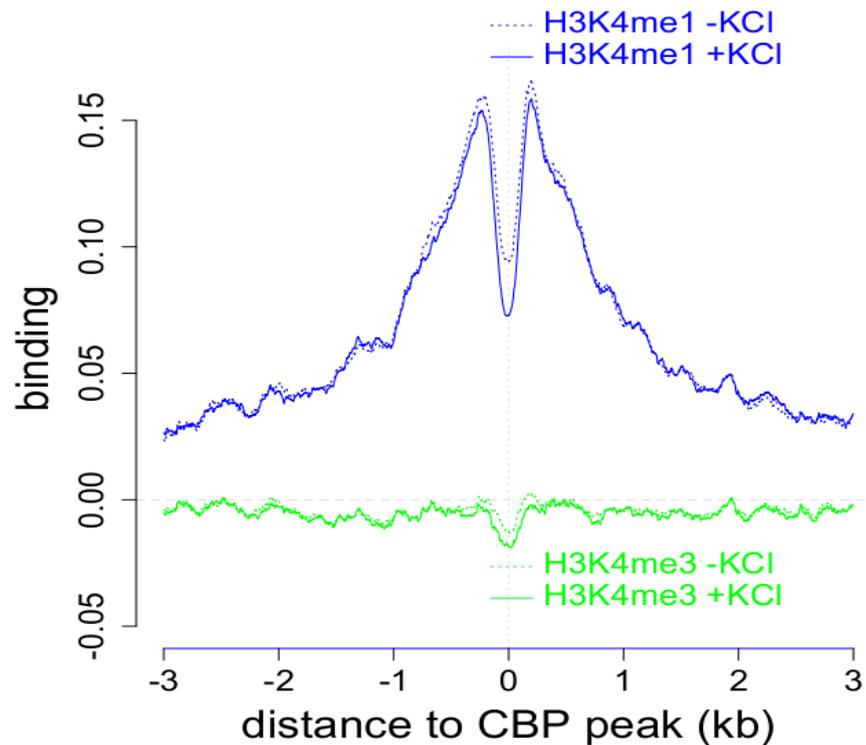
$$\tau_{decay} = \tau_{find} + \tau_{bp} L$$

$$H(x, t) = \frac{k\kappa}{\mu_x(\mu_x - \lambda)} (e^{-\lambda x} - e^{-\mu_x x}) \times e^{-\mu t}$$

$$E(x) = \sqrt{\frac{\pi}{2\lambda}} \frac{\gamma k}{\lambda} e^{-\delta^2/2\lambda - \lambda x^2/2} i \left[ \text{erf}\left(\frac{\delta i - \lambda i x}{\sqrt{2\pi}}\right) - \text{erf}\left(\frac{\delta i}{\sqrt{2\lambda}}\right) \right]$$

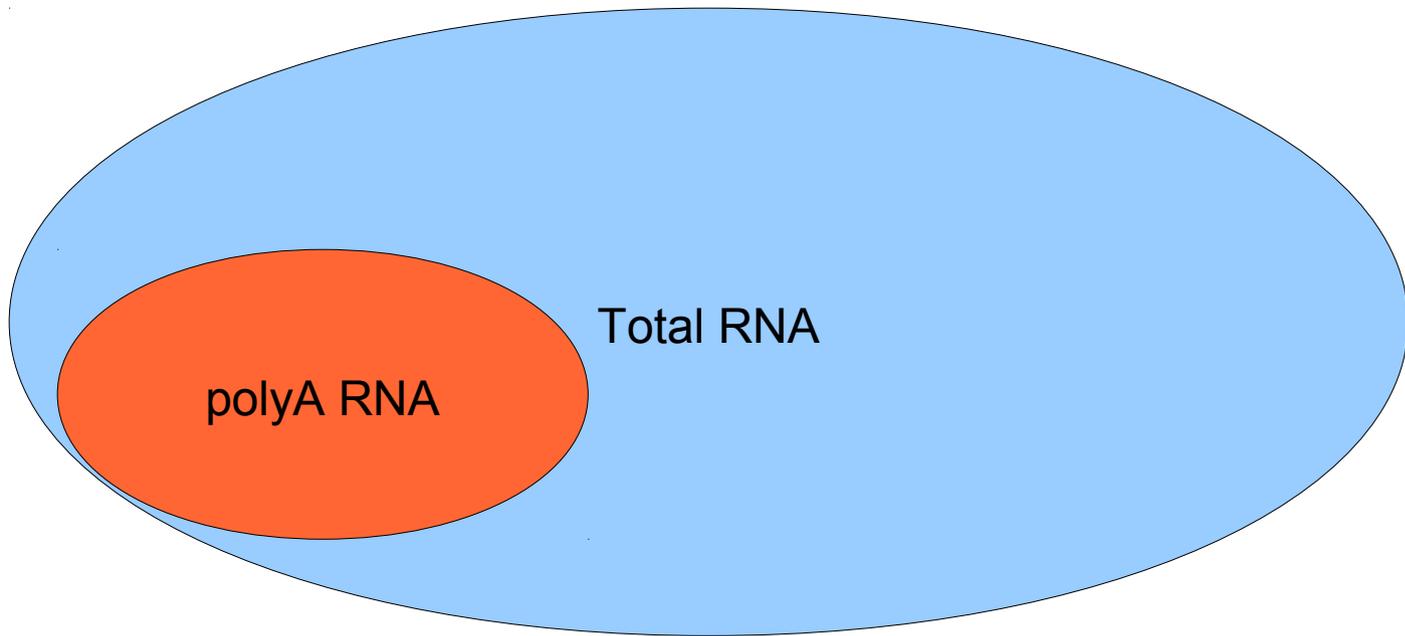
# How abundant are eRNAs compared to mRNAs?

- Identify **all** transcripts in the genome
  - Wavelet-based algorithm for *de novo* detection of transcribed regions accounts for 99.8% of reads
    - Annotated RNAs ~ 98.3%
    - eRNAs ~ 0.02%
      - 1 in 10,000 reads is an eRNA read
      - mRNAs ~100 times more abundant



# polyA tail is added to messenger RNAs (mRNAs)

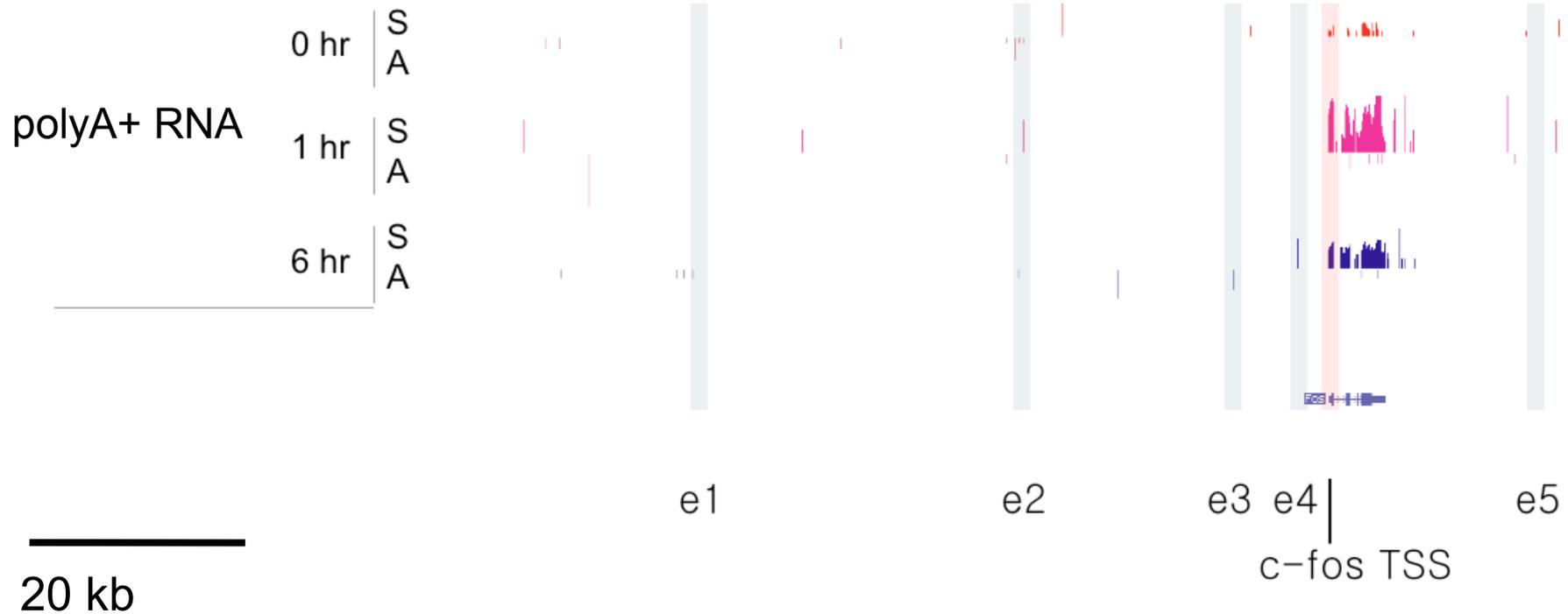
ACGUUUGUACCUAGCUAGCUUACGAG AAAAAAAAAAAAAAAAAAAAAA



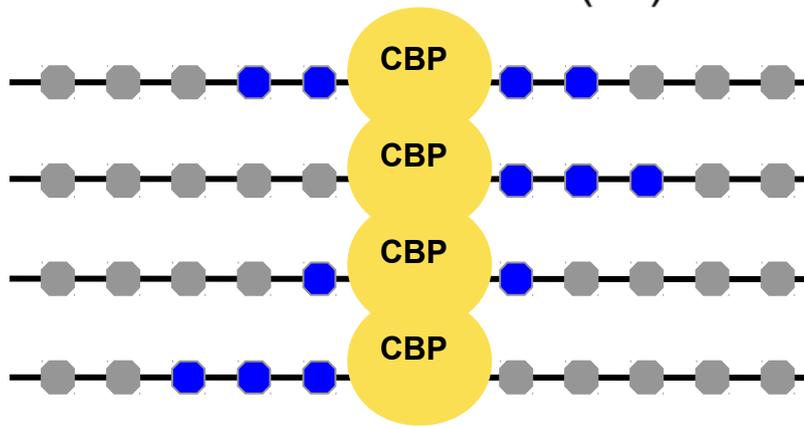
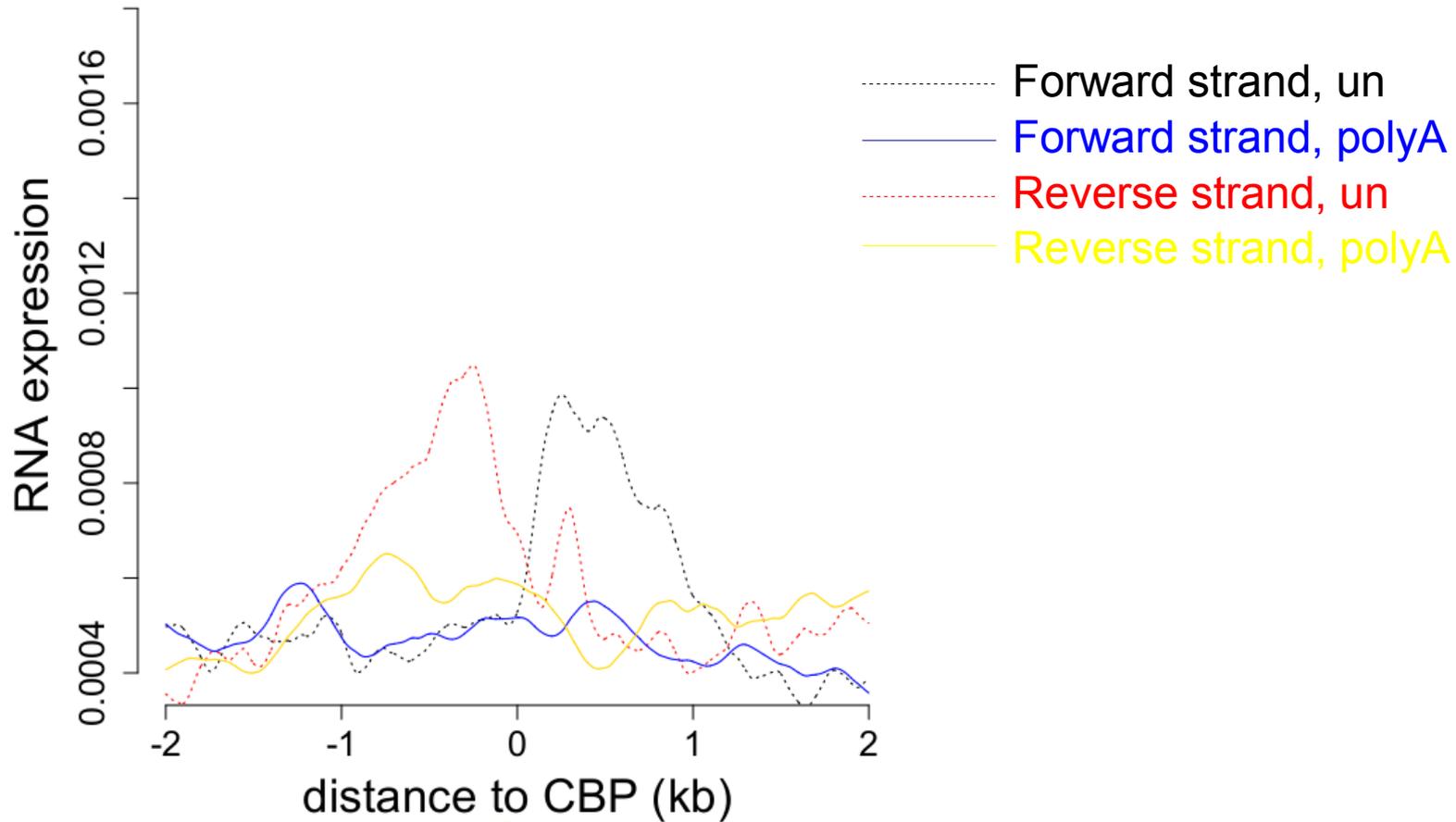
# Transcription of mRNA at the *fos* locus

ACGUUUGUACCUAGCUAGCUUACGAG

AAAAAAAAAAAAAAAAAAAAAAAA

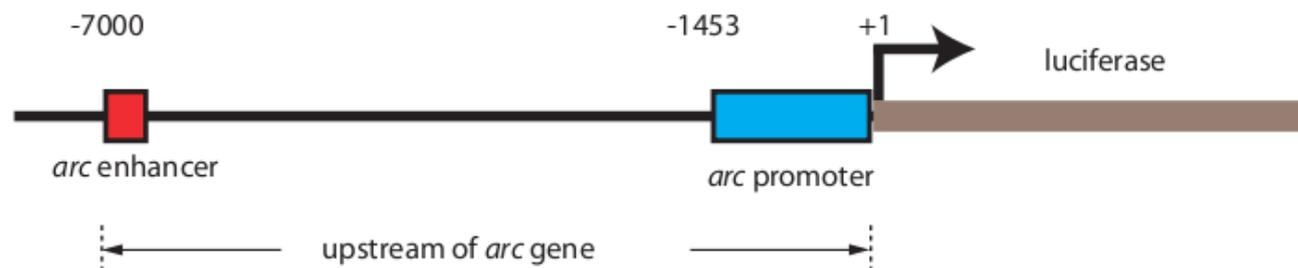


# eRNAs are not polyadenylated



We identified ~12,000 activity-dependent enhancers throughout the genome

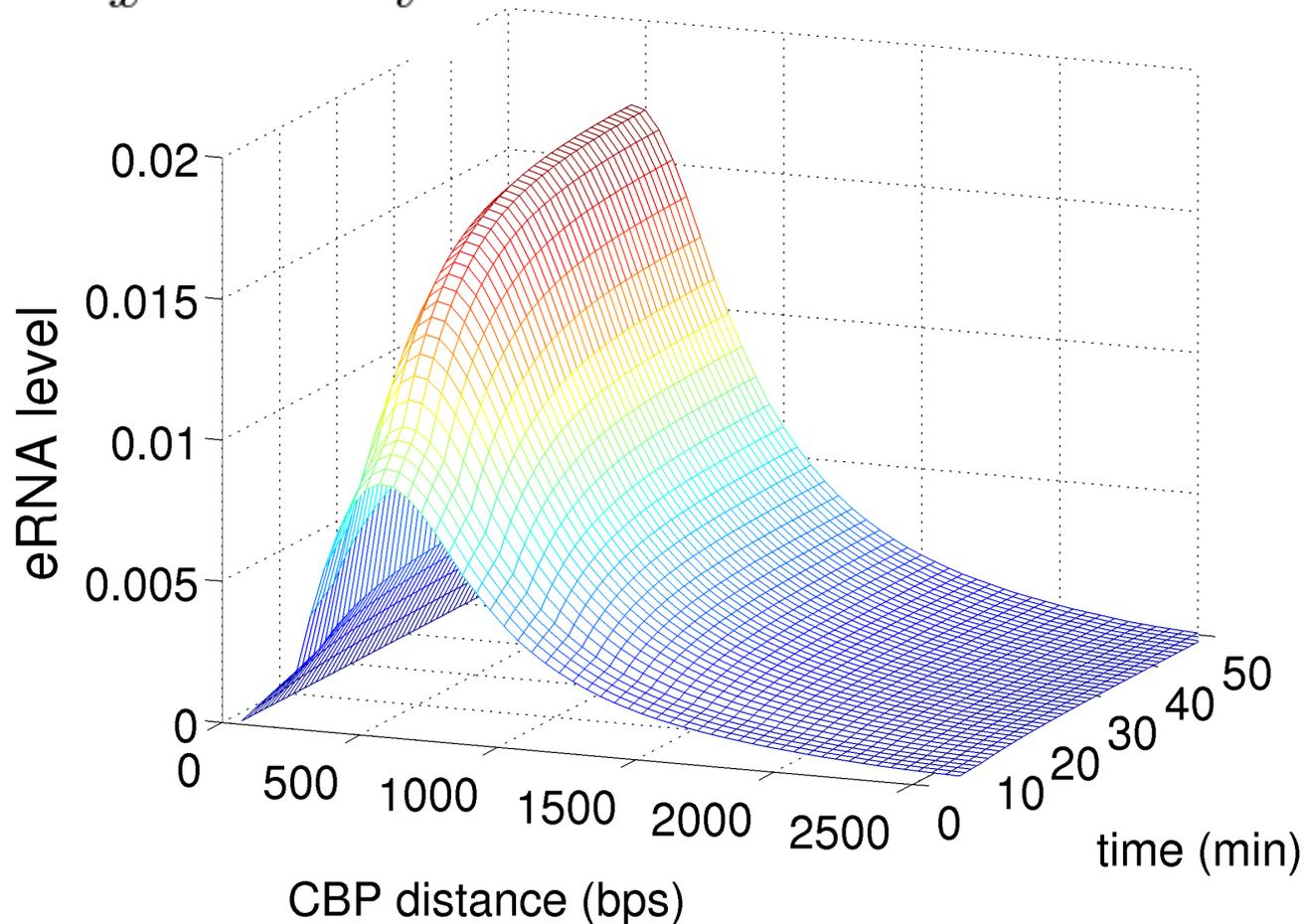
- **CBP** peak
- **High** levels of flanking **H3K4me1**
- **Low** levels of **H3K4me3**
  - 8/8 tested activity-dependent enhancers were validated using a luciferase assay



# A PDE for eRNA levels

$$\frac{\partial P}{\partial x} + \frac{\partial P}{\partial t} = k(x, t) - \lambda_x P - \lambda_t P$$

$$\frac{\partial E}{\partial x} + \frac{\partial E}{\partial t} = \gamma P(x, t) - \delta_x x E - \delta_t t$$



## Master Equation (**ME**) description

$$\frac{dP_j}{dt} = \sum_i W_{ij} P_i(t) - W_{ji} P_j(t)$$

$P_j$  - **Probability** of having j molecules

$W_{ij}$  - **Transition rate** from i to j